Chapter 5

Are stock prices driven by the volume of trade? Empirical analysis of the FT30, FT100 and certain British shares over 1988–1990

CHRIS G. ROGERS, STEPHEN E. SATCHELL,
AND YOUNGJUN YOON

ABSTRACT

This chapter explains how volume of trade appears to influence the log-return distribution of assets. To compensate for this is rather difficult. One strategy if you wish to predict prices is to consider the joint distribution of price and volume and derive the marginal distribution of price. This will be typically more fat-tailed than the normal. One can base confidence intervals on the predicted price based on the marginal distribution which will be smaller than under normality. The point estimate of price should be the same. A simpler alternative is to work with the conditional distribution of price given volume. To predict prices you can derive a confidence internal, but this will depend upon predictions of tomorrow's volume which may prove rather troublesome.

5.1 INTRODUCTION

For many years both financial economists and statisticians have been concerned with describing the behaviour of stock prices. The price changes in a stock market can be regarded as a result of the influx of new information into the market and of the re-evaluation of existing information. At any point in time there will be many items of information available. Thus, price changes between transactions will reflect the interactions of many different items of information. For example, in the prediction of price changes the difficulty comes from the uncertain arrival of new information as well as the random quantity of information at each point of the time series under study. Even though there is a remarkable discrepancy between the concepts of behaviour of stock prices held by professional stock market analysts, on the one hand, and by academics on the other, the form of the distribution of stock returns is important to both groups because it is a crucial assumption for mean-variance portfolio theory, theoretical models of capital asset prices, and the prices of contingent claims. In this chapter we examine the distribution of daily and weekly logarithmic returns of the FT100, FT30 and the firms that make up the FT30 over the period of 1988 to 1990. We uncover the usual results found by authors working with American data, namely that logarithmic returns measured either daily or weekly do not look normally distributed.

We then briefly discuss the literature that relates the price distribution to the volume of shares traded, a topic which has been examined in great detail by financial economists. The contribution of this chapter is to use volume, rather than time, as the forcing variable in our stochastic process for prices. Based on this assumption that business activity (volume) is driving the price and not time, we 'change the clock' of our process and re-evaluate the distribution of logarithmic returns when a certain volume of trade has elapsed, equal to the average weekly volume. This brings about a significant change in the distribution. It now appears much more normal and adds evidence to the hypothesis that share prices follow a subordinated log-normal process where the conditioning variable is volume. In Section 5.2 we present a review of the existing literature and the mathematical framework. In Section 5.3 we discuss normality testing, stock price indices and the price-volume relationship. In Section 5.4 we present our conclusions. We include definitions of the different normality tests in an appendix.

5.2 EARLY RESEARCH

Past studies of time series of prices at short intervals on a speculative market such as that for corporation shares, indices or futures on commodities are usually compatible with the log-normal random walk model which we shall describe next. We shall present this model in its continuous time version, the form in which it is currently most popular in financial economics. We assume that $s(t)$, the price of the asset at time t, is generated by

$$\partial s(t) = \alpha(t, s)s(t)dt + \sigma(t, s)s(t)dW(t) \qquad (1)$$

where α and σ represent instantaneous mean and volatility, respectively, and

$W(t)$ is standard Brownian motion (BM). The use of equation (5.1) is based on the hypothesis that the continuous Brownian motion is followed during periods between transactions and during periods of exchange closure, even though prices cannot be observed in such intervals. It is well known that, if $\alpha(t,s) = \alpha$ and $\sigma(t,s) = \sigma$ where α and σ are constant, equation (5.1) has the solution

$$s(t) = s(0)\exp[(\alpha - \tfrac{1}{2}\sigma^2)t + \sigma(W(t) - W(0))] \tag{5.2}$$

and that in the logarithmic form,

$$\ln\left(\frac{s(t)}{s(t-1)}\right) = (\alpha - \tfrac{1}{2}\sigma^2) + \sigma(W(t) - W(t-1)) \tag{5.3}$$

We see from equation (5.3) that $\ln(s(t))$ follows a random walk with drift and that errors are *i.i.d.* $N(0,\sigma^2)$, i.e.

$$\ln s(t) = \ln s(t-1) + (\alpha - \tfrac{1}{2}\sigma^2) + \xi(t) \tag{5.4}$$

where

$$\xi(t) = \sigma(W(t) - W(t-1)) \sim N(0,\sigma^2)$$

The increments in the price process are stationary in the mean and independent. If the mean is zero, this is exactly the random walk model. Here, the price changes are not absolute price changes but changes in the logarithmic prices which are independent of one another because stock market investors are interested in proportionate changes in the value of stocks.[1] Henceforth we will use the notation $S(t) = \ln(s(t))$ and $\Delta S(t) = \ln(s(t)) - \ln(s(t-1))$.

Besides empirical realism, the random walk model has a theoretical basis. If price changes are predictable, then alert speculators can make money until these opportunities are removed. Based on this argument, the efficient markets hypothesis implies that security prices reflect all publicly available information. This was first shown by Bachelier in 1900, when he derived the diffusion equation of a random walk model for security from a condition that speculators should receive no information from past prices. Later, Kendall (1953) confirmed that each period's price change was not significantly

[1]The reasons for using changes in logarithmic price is well explained in *E*. Fama (1965): the change in logarithmic price is the yield, with continuous compounding, from holding the security for that day; taking logarithms neutralizes most of the price level effect since the variability of simple price changes is an increasing function of the price level; for changes less than $\pm 15\%$ the change in logarithmic price is very close to the percentage price change.

correlated with the preceding period's price change nor with the price change of any earlier period. While Kendall worked with serial correlations for each series separately, Osborne (1959) worked with ensembles of price changes, which appeared to be approximately normally distributed with a standard deviation proportional to the square root of the length of the period. This proportionality of the standard deviation of price differences to the square root of the differencing period is a characteristic of a random walk and had been pointed out much earlier by Bachelier (1900). In Bachelier's case, however, the differences were arithmetic, while in Osborne's they were logarithmic.

Normality of asset returns was a popular assumption in investigations of investors' behaviour. For this reason in the early stage of stock market study, the normal distribution was considered as a good description of stock market returns. The normal distribution arises in many stochastic processes involving large numbers of independent variables. The traditional justification of log-normality is based on a multiplicative version of the Central Limit Theorem because the change of returns within a certain interval is a product of each individual transaction change of returns. The normal distribution has special virtues; it is linked with the classical Central Limit Theorem; it is stable, meaning any linear combination of independent normals is itself normal; and it is analytically tractable.

In the general theory of random walks the form or shape of the distribution need not be specified. Previous authors (Clark, 1973; Epps and Epps, 1976; Fama, 1963; Mandelbrot, 1963; Tauchen and Pitts, 1983) have found that the price changes $\Delta S_t = \ln(s(t)) - \ln(s(t-1))$, however independent, are not normally distributed. Instead of having the normal shape, which would be the case if the components in ΔS_t were almost independent and almost identically distributed, ΔS_t is consistently more leptokurtic (is more peaked and has fatter tails) than normality indicates. Also, several authors have noted that the nature of the return distribution may change as the period length changes. Assuming that the distribution is stationary with finite mean and variance, this would imply that the leptokurtosis observed in the distribution of daily returns will become less severe as we increase the interval of measurement. This is because we are adding together independent increments with a finite variance which allows an application of the Central Limit Theorem. However, conditions sufficient for the Central Limit Theorem are not met by the influences which make up ΔS_t. The standard Central Limit Theorem holds only when the number of random variables being added is at least non-stochastic; in the case of speculative markets, this restriction may be violated. The number of individual effects added together to give the price change during a certain interval is random, making the standard Central Limit Theorem inapplicable. Although this does not exclude the possibility of normal distributions from our consideration, it gives an insight into why non-normality may arise in practice.

Two responses to these empirical findings have evolved. The first centred on the use of stable Paretian distributions (see Fama, 1963, and Mandelbrot, 1963). We shall not discuss the stable distribution in this paper but look directly at the second approach, the use of subordinated stochastic processes. The hypothesis is that the distribution of price changes is subordinate to a stochastic process generated from a mixture or combination of distributions. The price series evolves at different rates during identical intervals of time where the variance of the distribution is itself a random variable. The different evolution of price series on different days is due to the fact that information is available to traders at a varying rate. Therefore, the distribution of price changes should be defined conditional on the information-generating process, so that the limit distribution of price changes is subordinate to some distribution. For example, if $P(t)$ is normal with stationary independent increments, and $T(t)$ has stationary independent positive increments with finite second moments which are independent of P, then the subordinated stochastic process $P(T(t))$ has stationary independent increments and the kurtosis of the increments of $P(T(t))$ is an increasing function of the variance of the increments of $T(t)$. Therefore, the introduction of any directing process makes the distribution of the increments of $P(T(t))$ only more leptokurtic. The limit distribution of a random sum of random variables which obey the Central Limit Theorem is asymptotically normal with random variance, or new terminology, subordinate to the normal distribution. Upton and Shannon (1979) found that the asymptotic tendencies of the return distribution are in agreement with the implications of the subordinated stochastic process approach rather than the stable Paretian distribution. Kon (1984) proposed a discrete mixture of normal distributions rather than a continuous mixture to explain the observed significant kurtosis (fat tail) and significant positive skewness[2] in the distribution of daily rate of returns for a sample of common stocks and indices. He found that the data could be well described by a mixture of normals, the actual number of normal distributions involved may vary across firms. Stationarity tests on the parameter estimates of the discrete mixture of normal distributions model revealed significant differences in the mean estimates that can explain the observed skewness in security returns. Significant differences in the variance estimates also can explain the observed kurtosis.

---

[2]There is some evidence indicating that the assumption of symmetric empirical distributions may be violated for certain phenomena, see Fielitz and Smith (1972) and Leitch and Paulson (1975).

5.3 TESTING NORMALITY IN THE INDIVIDUAL STOCKS

We next describe our data. We collected the data for two Financial Times indices and 30 individual British companies for the period of 1/1/88-31/12/90. We chose this period to avoid any difficulty due to distributional shifts pre and post the October 1987 crash. We started at 1/1/88 to allow some of the short-run perturbations of the crash to settle down. It is an interesting question as to whether there has been a distributional shift before and after the crash, but we shall not address it in this chapter. Two indices, FT-SE100 and FT30, were chosen since they have distinct features, which will be explained later in this section. The 30 companies[3] chosen are the constituents of the FT30 index. Three different time intervals, daily, weekly and fortnightly, were used for the normality tests. For the weekly and fortnightly data, Friday was chosen as the day to measure returns from.

The goodness-of-fit tests of normality[4] are based on the skewness statistic $\sqrt{b_1}$, the kurtosis statistic b_2, a joint test using $\sqrt{b_1}$, and b_2 (Bera–Jarque Test), and definitions are given in the Appendix. Where these tests are used, some care should be taken; they are asymptotic tests and can only be justified by a relatively large sample size, also the tests are sensitive to outliers (e.g. unusually large deviations perhaps caused by stock crashes) (see Spanos, 1986). To cover this weakness, Klein's method is added, which is based on the comparison of observed frequencies with theoretical frequency within quantile limits. Also, we reported the results from the Kolmogrov-Smirnov test.[5] Detailed descriptions of these tests are in the Appendix.

We apply these test procedures to the 30 constituent companies of the FT30. The results are generated in Table 5.1 for daily, Table 5.2 for weekly, and Table 5.3 for fortnightly. Only 12 companies out of 30 satisfied the five test statistics used for normality based on the fortnightly data at $\alpha = 0.05$, 11 for the weekly data. None of the daily data satisfy all test statistics. This leads us to reject the normal distribution of the stock returns traded in the London Stock Exchange. Most of them failed to satisfy the kurtosis statistic b_2, especially in the daily

[3]The weekly result for Beecham is omitted because of insufficient data since it was merged into SmithKline Beecham during the period.

[4]Tests for departures from normality can be divided into parametric and non-parametric tests depending on whether the alternative is given a parametric form or not. Several works on the power of tests for normality reported that b_2 and $\sqrt{b_1}$ are generally preferred, see (see D'Agostino and Pearson, 1973; Gastwirth and Owens, 1977; Saniga and Miles, 1979; Shapiro and Wilk, 1965; Shapiro, Wilk and Chen, 1968).

[5]Since the Kolmogrov–Smirnov test requires the complete specification of the null distribution, the mean and variance of the specified simple normal hypothesis were taken as the (known) mean and variance of the actual alternative distribution. This will cause a slight mismeasurement akin to using normal tables for the t test. Our smallest sample is 156 observations, which renders this effects quite negligible.

Table 5.1 $(S_2 - S_1)$, $(S_3 - S_2)$, $(S_4 - S_3)$, ... (daily price)

	B–J	$\sqrt{b_1}$	b_2	Klein's χ^2 (27)	K–S
Allied-Lyons	331.2	0.505	6.089	110.7	0.057
Asda-MFI	1026.3	−0.391	8.650	148.8	0.079
BICC	197.2	−0.196	5.470	82.74	0.048
BOC	109.1	0.048	4.857	150.4	0.051
BTR	1108.8	−0.649	8.485	90.70	0.055
Beecham	14.39	0.161	3.878	44.28	0.038
Blue Circle	64.31	0.224	4.356	96.56	0.058
Boots	156.1	−0.042	5.223	58.79	0.058
British Airways	117.1	−0.071	4.921	199.6	0.067
British Gas	19.79	0.058	3.784	77.19	0.068
British Petrol	120.4	0.392	4.790	64.71	0.062
British Telecom	48.61	0.202	4.174	78.98	0.059
Cadbury	1224.8	1.064	8.857	142.2	0.092
Courtaulds	353.8	0.414	6.245	92.00	0.062
Gen. Electric	136.4	0.222	5.032	168.4	0.083
Glaxo	27.05	0.037	3.923	40.13	0.038
Grand Metro.	40.68	−0.123	4.109	43.98	0.050
GKN	134.7	−0.482	4.828	105.9	0.064
Guinness	319.8	0.601	5.948	129.8	0.065
Hanson Trust	76.68	−0.042	4.557	90.85	0.060
Hawker Siddeley	1782.9	−1.052	10.22	102.2	0.060
ICI	312.7	−0.696	5.824	55.07	0.045
Lucas	781.1	−0.356	7.925	113.8	0.067
M & S	130.8	0.059	5.033	122.2	0.072
Nat. West. Bank	371.3	0.095	6.426	108.2	0.063
P & O	178.5	−0.024	5.378	75.62	0.044
Royal Ins.	81.80	−0.086	4.601	129.5	0.061
Tate & Lyle	43.30	0.087	4.159	134.7	0.077
Thorn-EMI	85.96	0.027	4.650	54.58	0.050
Trusthouse	6.945	0.063	3.452	79.61	0.063

$\chi^2(2) = 5.99$	at $\alpha = 0.05$	and	$\chi^2(2) = 9.21$	at $\alpha = 0.01$
$-0.23 \leq \sqrt{b_1} \leq 0.28$	at $\alpha = 0.05$	and	$-0.403 \leq \sqrt{b_1} \leq 0.403$	at $\alpha = 0.01$
$2.51 \leq b_2 \leq 3.57$	at $\alpha = 0.05$	and	$2.37 \leq b_2 \leq 3.98$	at $\alpha = 0.01$
$KS \leq 1.36/\sqrt{N}$	at $\alpha = 0.05$	and	$KS \leq 1.63/\sqrt{N}$	at $\alpha = 0.01$
$\chi^2(27) = 40.1$	at $\alpha = 0.05$	and	$\chi^2(27) = 47.0$	at $\alpha = 0.01$

data, while the symmetry looked quite reasonable. The tables suggest that the length of interval is closely related to the kurtosis of stock returns. Weekly versus daily of not rejecting the null hypothesis is 14 versus 1. This indicates that daily information arrivals fluctuate relatively more than weekly ones. This phenomenon becomes more apparent in our fortnightly data of Table 5.3. Only nine companies failed to satisfy the kurtosis statistic, and in general normality is improved. However, there exists a difficulty in symmetry due to the insufficient data since the fortnightly data reduced the sample size. This evidence is consistent with the findings of other authors.

Table 5.2 $(S_2 - S_1)$, $(S_3 - S_2)$, $(S_4 - S_3)$, ... (weekly price)

	B–J	$\sqrt{b_1}$	b_2	Klein's χ^2 (12)	K–S
Allied-Lyons	30.28	0.664	4.702	41.04	0.090
Asda-MFI	28.29	−0.519	3.416	11.29	0.051
BICC	1.632	−0.140	3.416	9.803	0.053
BOC	1.701	0.091	3.478	15.01	0.067
BTR	8.118	−0.339	3.889	9.311	0.041
Beecham	–	–	–	–	–
Blue Circle	5.572	0.431	3.337	9.783	0.057
Boots	0.853	0.091	2.687	11.83	0.042
British Airways	3.720	0.110	3.724	16.19	0.049
British Gas	12.88	0.231	4.330	17.69	0.053
British Petrol	7.801	0.421	3.702	21.96	0.063
British Telecom	7.816	0.271	3.953	10.93	0.057
Cadbury	388.3	1.838	9.799	27.93	0.107
Courtaulds	0.777	−0.162	3.119	8.831	0.030
Gen. Electric	0.313	−0.087	3.133	4.739	0.042
Glaxo	3.354	0.242	3.531	9.253	0.040
Grand Metro.	1.284	−0.122	3.372	9.937	0.046
GKN	1.225	−0.203	3.152	20.08	0.041
Guinness	3.990	0.244	3.612	14.62	0.049
Hanson Trust	0.897	−0.185	2.980	10.12	0.050
Hawker Siddeley	44.45	−0.792	5.080	25.87	0.050
ICI	10.94	−0.254	4.194	12.27	0.065
Lucas	49.08	−0.424	5.614	19.70	0.062
M & S	0.452	0.125	2.913	8.444	0.045
Nat. West. Bank	31.67	0.543	4.922	8.595	0.064
P & O	1.048	−0.115	3.329	7.529	0.038
Royal Ins.	6.284	0.322	3.743	9.471	0.044
Tate & Lyle	6.696	0.353	3.730	38.93	0.050
Thorn-EMI	2.266	−0.293	2.928	9.243	0.047
Trusthouse	2.809	0.302	3.258	8.897	0.061

$\chi^2(2) = 5.99$	at $\alpha = 0.05$	and	$\chi^2(2) = 9.21$	at $\alpha = 0.01$
$-0.23 \leq \sqrt{b_1} \leq 0.28$	at $\alpha = 0.05$	and	$-0.403 \leq \sqrt{b_1} \leq 0.403$	at $\alpha = 0.01$
$2.51 \leq b_2 \leq 3.57$	at $\alpha = 0.05$	and	$2.37 \leq b_2 \leq 3.98$	at $\alpha = 0.01$
$KS \leq 1.36/\sqrt{N}$	at $\alpha = 0.05$	and	$KS \leq 1.63/\sqrt{N}$	at $\alpha = 0.01$
$\chi^2(12) = 21.0$	at $\alpha = 0.05$	and	$\chi^2(12) = 26.2$	at $\alpha = 0.01$

5.3.1 Indices and their distributions

There has always been the need for a summary statistic to measure stock market performance, since the aggregate performance of the stock market is an indicator of the state of the overall economy and monitoring the performance of the market provides a powerful source of information for investment decisions. As a summary of the direction and extent of average changes of stock prices, stock price averages[6] or indices provide a convenient way to summarize general market movements. They are constructed by

Table 5.3 $(S_2 - S_1)$, $(S_3 - S_2)$, $(S_4 - S_3)$, ... (fortnightly price)

	B–J	$\sqrt{b_1}$	b_2	Klein's χ^2 (3)	K–S
FT30 Index	0.215	−0.104	2.849	0.697	0.034
FT100 Index	0.310	−0.047	2.705	1.973	0.034
Allied-Lyons	3.779	0.382	3.760	4.391	0.066
Asda-MFI	56.55	−1.179	6.439	8.004	0.085
BICC	1.463	−0.244	2.540	4.654	0.065
BOC	0.035	0.020	3.096	1.635	0.062
BTR	1.637	−0.325	3.282	3.762	0.083
Blue Circle	1.475	0.299	2.690	3.171	0.072
Boots	0.600	−0.150	2.692	1.106	0.046
British Airways	0.788	−0.245	3.026	2.620	0.067
British Gas	0.093	0.066	2.489	2.040	0.064
British Petrol	1.552	0.203	3.558	2.007	0.071
British Telecom	0.690	−0.198	2.765	3.093	0.046
Cadbury	63.33	1.453	6.322	15.90	0.133
Courtaulds	3.023	−0.435	2.586	5.477	0.064
Gen. Electric	0.023	−0.037	2.962	3.438	0.073
Glaxo	0.263	−0.034	2.723	2.331	0.051
Grand Metro.	0.593	−0.115	2.640	3.439	0.040
GKN	0.553	−0.201	3.086	0.557	0.036
Guinness	16.39	−0.541	4.967	2.343	0.051
Hanson Trust	1.455	0.174	2.428	1.160	0.069
Hawker Siddeley	24.75	−1.110	4.638	14.67	0.1169
ICI	4.045	−0.537	3.299	2.025	0.048
Lucas	8.645	−0.117	4.614	3.521	0.082
M & S	1.590	0.341	3.154	4.609	0.070
Nat. West. Bank	7.491	0.137	4.493	7.115	0.088
P & O	2.644	−0.341	2.411	5.513	0.070
Royal Ins.	1.453	0.248	3.447	3.745	0.081
Tate & Lyle	0.241	0.010	3.271	1.586	0.062
Thorn-EMI	2.069	−0.265	2.404	8.086	0.098
Trusthouse	2.508	0.409	3.316	1.994	0.072

$\chi^2(2) = 5.99$	at $\alpha = 0.05$	and	$\chi^2(2) = 9.21$	at $\alpha = 0.01$
$-0.23 \leq \sqrt{b_1} \leq 0.28$	at $\alpha = 0.05$	and	$-0.403 \leq \sqrt{b_1} \leq 0.403$	at $\alpha = 0.01$
$2.51 \leq b_2 \leq 3.57$	at $\alpha = 0.05$	and	$2.37 \leq b_2 \leq 3.98$	at $\alpha = 0.01$
$KS \leq 1.36/\sqrt{N}$	at $\alpha = 0.05$	and	$KS \leq 1.63/\sqrt{N}$	at $\alpha = 0.01$
$\chi^2(3) = 7.81$	at $\alpha = 0.05$	and	$\chi^2(3) = 11.3$	at $\alpha = 0.01$

[6]Even though the price average such as the Dow Jones Average has been widely quoted, it is criticized for the following reasons; splitting bias in which the divisor for the average has to be adjusted regularly to accommodate splits and it implicitly puts more weight to the stocks that remain unsplit, anti-growth bias since growth stocks split more than non-growth stocks, arithmetic mean bias which gives equal weight to equal absolute rather than the percentage changes in stock prices, etc.

sampling; selecting some manageable number of stocks to act as a proxy for the universe of all stocks. The sample is then weighted in some way, assigning different levels of importance to various component stocks. Next, the weighted sample is averaged, arithmetically or geometrically, to produce a single summary number. If it is a price index, the weighted average of the sample is further divided by a constant to relate it to an arbitrary but intuitively meaningful base value.

Indices are usually weighted by the number of shares outstanding for each stock multiplied by the price of the stock. These capitalization weights reflect relative weights based on each company's capitalization. FT-SE 100, S&P 500 and NYSE indices belong to this category. The value weight indicates changes in the aggregate market value of stocks. Thus, changes in general market value are more reflected in these indices for studies of relationships between stock prices and other things in the national economy with more importance to a relatively few large companies.

The FT-Actuaries Share Indices are weighted arithmetic averages of the price relative; the weights used being the initial capitalization, subsequently modified to maintain the continuity when capital and constituent changes occur. They are derived to show the longer-term changes associated with the value of a portfolio over time, although still reflecting day-to-day movements. The Financial Times–Stock Exchange 100 Share Index generally represents the 100 largest companies by market capitalization. The choice of 100 shares was to hit the balance between the practical difficulty of collecting around 750 shares on a real-time basis needed to turn the All Share Index into a real-time index, and yet having sufficient cover of the market to closely follow the movement of the All Share Index. It mirrors the movement of a typical institutional portfolio. A base figure of 1000 was chosen to make the index more tradeable on the futures or options markets as a high base contract figure usually produces whole number changes every day. As a preliminary, we carried out the same tests reported in Tables 5.1 and 5.2 for the FT100 from 1/1/88 to 31/12/90. The results are presented in Table 5.3. We delay the discussions of results until after an analysis of the FT30.

The Financial Times Ordinary Share Index (FT30) is the geometric average of 30 securities on an unweighted or unit-weighted basis and aims to show short-term movements in the market. The geometric average[7] involves the product of n numbers of component stocks and the nth root of that product which preserves the integrity of successive upward and downward percentage changes in stock prices. The Index is calculated on a 'real-time' basis from the

[7]It has an unavoidable downward bias; the geometric mean is always less than the arithmetic mean of the same numbers.

start of trading at 9 a.m. A closing Index is produced soon after 5 p.m. on the basis of prices collected at the close down of the Stock Exchange SEQA system. Since the Index is unweighted the calculation is simple. Also, it is sensitive because it is based on heavily traded blue chip shares which are the first to respond to any changes in stock market sentiment. The equal-weight indices may be more appropriate for indicating movements in the prices of typical or average stocks and are better indicators of the expected change in the prices of stocks selected at random since relatively small companies are more sensitive to economic trends. Thus it has been widely followed up to the advent of the FT100.[8] The 30 constituents are carefully selected so as to form a representative spread across British industry and commerce. The number 30 was originally chosen as the best compromise between ease and spread of calculation, on the one hand, and, on the other, the need to avoid too large an influence by freak movements in one or two individual share prices. Its constituents are heavy industry (6), textiles (4), motor & aviations (3), electrical manufacturer and radio (3), building materials (3), food, drink and tobacco (6), retail stores (2), financial institutions (2), miscellaneous (1).

Since the FT30 is a geometric average index, it is easier to make allowances for capital changes, and to replace constituents, without the need for rebasing. Moreover, it damps down the impact of large rises in individual constituents. Despite its advantages it tends to bias the Index downwards over the longer term. This is partly a purely mathematical effect,[9] but it also reflects the way that poorly performing constituents enter into the Index. Therefore, the FT30 Index should not be used as a long-term measure of market levels or as a yardstick for portfolio performance. It should be used for the purpose for which it was precisely designed, as a sensitive indicator of the mood of the market, originally from day-to-day and now from hour-to-hour.

We present the results of our normality tests on the indices in Table 5.4 and Table 5.5. The normality tests of the FT100 and the FT30 show similar conclusions to the individual stocks as before. Each day of the trading days from Monday to Friday was tested. Only Monday and Friday for the FT30 and Wednesday for the FT100 follow a normal distribution at the 5% level. There is no striking improvement in the FT30 compared with individual shares while the FT100 looks reasonably normal. In fact, the FT100 index satisfies all the tests at the 1% level.

A possible problem with these datasets is the presence of serial correlation. We investigated the FT30 and FT100 daily and weekly data. The only

---

[8]The Index has been used since 1935. About a quarter of its constituents have remained in the 30 throughout the period.

[9]This is by Jensen's Inequality, $E[g(X))] \leq g(E[X])$ where h is convex.

Table 5.4 Tests for normality of FT100

	Monday	Tuesday	Wednesday	Thursday	Friday
Observations	157	157	156	156	157
Bera–Jarque χ^2 (2)	**3.3529**	**1.0450**	**2.2581**	6.6615	**4.3629**
$\sqrt{b_1} = m_3/m_2^{3/2}$	**0.0610**	**−0.1771**	**−0.2545**	**−0.2496**	**0.2709**
$b_2 = m_4/m_2^2$	3.7078	**3.1877**	**3.3010**	3.8845	3.6146
Klein's χ^2 (12)	**15.8752**	21.9950	**6.6425**	**7.6476**	**10.3683**

Notes: Details and definitions of the notation are given in the Appendix.
Here and in Table 5.6 the bold numbers indicate not rejecting the null hypothesis of normality.

Table 5.5 Tests for normality of FT30

	Monday	Tuesday	Wednesday	Thursday	Friday
Observations	157	157	156	156	157
Bera–Jarque χ^2 (2)	**1.8002**	6.6907	6.3208	9.9176	**2.8183**
$\sqrt{b_1} = m_3/m_2^{3/2}$	**−0.0247**	0.3476	**−0.2507**	−0.2845	**0.2448**
$b_2 = m_4/m_2^2$	**3.5239**	3.7390	3.8528	4.1008	**3.4403**
Klein's χ^2 (12)	**6.7854**	**11.0823**	**7.1537**	**11.6313**	**18.1187**

$\chi^2(2) = 5.99$	at $\alpha = 0.05$	and	$\chi^2(2) = 9.21$	at $\alpha = 0.01$
$-0.23 \leq \sqrt{b_1} \leq 0.28$	at $\alpha = 0.05$	and	$-0.403 \leq \sqrt{b_1} \leq 0.403$	at $\alpha = 0.01$
$2.51 \leq b_2 \leq 3.57$	at $\alpha = 0.05$	and	$2.37 \leq b_2 \leq 3.98$	at $\alpha = 0.01$
$\chi^2(12) = 21.0$	at $\alpha = 0.05$	and	$\chi^2(12) = 26.2$	at $\alpha = 0.01$

significant autocorrelations found were the first lagged variables for daily data in both cases, in fact the coefficient for the FT30 was estimated at 0.083 with a t-level of 2.226 and for the FT100 0.084 with a level of 2.257. For stationary processes, the behaviour of test statistics based on functions of the first four moments will not be influenced by autocorrelation under the null at least asymptotically. However, the power of the tests may well be affected. Since the alternative to normality is not specified, this seems a problem for further research. One could fit an Edgeworth-type family under the alternative and calculate the power function as a function of the autocorrelation coefficient, but we have not done this. If the autocorrelation processes were non-stationary this would influence our test statistics, but then the question of testing for normality becomes meaningless.

Log-normal and log-stable distributions have multiplicative stability but not additive stability. Strictly speaking, if individual asset returns are log-normally (or log-stably) distributed, FT100 returns must have some other distribution while FT30 has a log-normal distribution. Thus if we believe the process has each share generated by equation (5.1), we might expect better results for normality for the FT30 than for the FT100. Against this, there is a possibility that adding more shares together, in the case of the FT100, will induce normality via central limit theorem results. The results in Tables 5.4 and 5.5 indicate that instability under addition, and the problem of changing weights, are not a practical concern under the conditions studied because there is little evidence that FT30 is better suited to a normal distribution than FT100.[10] The practical importance of these complications is an empirical question.

5.3.2 The price–volume relationship

The price–volume relation is critical to the debate over the empirical distribution of stock prices. According to Karpoff (1987), the variance of the daily price change and the mean daily trading volume depend upon three factors: (1) the average daily rate at which new information flows to the market, (2) the extent to which traders disagree when they respond to new information, and (3) the number of active traders in the market. In general, volume is positively related to the magnitude of the price change and, in equity markets, to the price change *per se*. Clark (1973) derives the positive relationship through randomness in the number of within-period transactions. The daily price change is the sum of a random number of within-day price changes. The variance of the daily price change is thus a random variable with a mean proportional to the mean number of daily transaction. Since the trading volume is related positively to the number of within-day transactions, so the trading volume is related positively to the variability of the price change. Another possibility, suggested by Tauchen and Pitts (1983), comes from the fact that the change in the market price on each within-day transaction or market clearing is the average of the changes in all of the traders' reservation prices. Assuming that there is a positive relationship between the extent to which traders disagree when they revise their reservation prices and the absolute value of the change in the market price, the price variability–volume relationship arises because the volume of trading is positively related to the extent to which traders disagree when they revise their reservation prices.

When sampled over fixed calendar intervals (e.g. days), rates of return turned to appear kurtotic compared to the normal distribution in the previous tests. Here, we can develop the explanation of price behaviour by incorporating volume into our consideration. Price–volume tests generally support the mixture of distributions hypothesis which implies that price data are generated by a conditional stochastic process with a changing variance parameter that can be proxied by volume. Osborne (1959) attempted to model

[10] In fact, there are 12 for FT30 versus 14 for FT100 significant entries in the tables.

the stock price change as a diffusion process with variance dependent on the number of transactions. This could imply a positive correlation between V and $|\Delta S|$, as later developed by Clark (1973), Tauchen and Pitts (1983), and Harris and Gurel (1986). In statistical terms, we are postulating a conditional distribution of ΔS, given V. If we assume a marginal distribution of V we know the joint distribution of ΔS and V, and if we integrate out V we have the marginal distribution of ΔS. This marginal distribution may well exhibit the characteristics discussed earlier.

Clark (1973) used trading volume as a measure of the speed of evolution from new information. The distribution of the increments of the price process would then have a distribution subordinate to that of the price changes on individual trades, and directed by the distribution.of trading volume. Trading volume is taken as an instrument for the true operational time, or an imperfect clock measuring the speed of evolution of the price-change process. Clark showed that the kurtosis has been very much reduced when price changes with similar volumes were considered. His method is to group by similar volume classes, treating each observation independently, not as time-series data. As long as there is no autocorrelation, his regrouping works. However, if there is any serial correlation, this method will be misleading. Epps and Epps (1976) have suggested that volume moves with measures of within-day price variability because the distribution of the transaction price change is a function of volume. The change in the logarithm of price can therefore be viewed as following a mixture of distributions, with transaction volume as a mixing variable. Tauchen and Pitts (1983) derived a bivariate normal mixture model of price and volume with a likelihood function based on the variance-components scheme. They also considered growth in the size of speculative markets; as the number of traders grows secularly over days, the variance of price changes declines monotonically while the mean volume of trading grows linearly with traders.

First, following Clark's (1973) method, the normalities of weekly FT30 and FT100 changes were tested conditional on the traded volume. Instead of grouping the samples within the same range of volume, the prices were

Table 5.6 Tests for normality conditional on volume

	FT-SE100 Index	FT-30 Index
Observations	157	157
Bera–Jarque χ^2 (2)	**1.6505**	**0.2492**
$\sqrt{b_1} = m_3/m_2^{3/2}$	**0.2270**	**0.2477**
$b_2 = m_4/m_2^2$	**2.5869**	**3.0129**
Klein's χ^2 (12)	**12.8490**	**11.0454**

Table 5.7 Price change at every equal amount of traded volume based on weekly averages

	B–J	$\sqrt{b_1}$	b_2	Klein's χ^2 (12)	K–S
FT30 Index	0.249	0.247	3.012	11.05	0.054
Allied-Lyons	9.573	0.203	4.187	15.06	0.059
Asda-MFI	0.310	−0.074	2.834	4.427	0.053
BICC	4.034	−0.400	3.073	19.26	0.037
BOC	0.958	−0.190	3.060	11.88	0.065
BTR	16.80	−0.513	4.258	15.73	0.061
Blue Circle	7.147	0.225	3.947	16.59	0.054
Boots	0.480	−0.121	2.872	5.296	0.044
British Airways	2.120	−0.286	2.958	8.973	0.045
British Gas	0.573	0.052	3.281	8.381	0.056
British Petrol	1.155	0.190	2.804	5.674	0.055
British Telecom	0.159	0.066	3.084	8.378	0.034
Cadbury	5.429	0.498	3.024	18.82	0.060
Courtaulds	16.75	−0.331	4.474	22.62	0.063
Gen. Electric	4.769	0.304	3.608	21.60	0.059
Glaxo	3.115	−0.317	2.721	10.53	0.051
Grand Metro.	2.130	−0.269	3.198	11.73	0.061
GKN	14.66	−0.688	3.659	23.09	0.068
Guinness	21.97	0.007	4.907	15.80	0.072
Hanson Trust	0.182	0.082	2.970	13.86	0.044
Hawker Siddeley	2.807	−0.329	3.022	6.110	0.043
ICI	0.663	−0.030	3.314	6.726	0.039
Lucas	39.75	−0.066	5.494	29.17	0.054
M & S	8.721	0.382	3.870	10.23	0.040
Nat. West. Bank	21.59	0.264	4.744	8.464	0.052
P & O	53.59	0.653	5.621	14.09	0.058
Royal Ins.	1.415	0.152	3.360	8.354	0.044
Tate & Lyle	2.661	−0.143	3.572	15.61	0.038
Thorn-EMI	0.179	0.081	2.960	9.308	0.044
Trusthouse	4.245	−0.098	3.789	10.66	0.041

$\chi^2(2) = 5.99$	at $\alpha = 0.05$	and	$\chi^2(2) = 9.21$	at $\alpha = 0.01$
$-0.23 \leq \sqrt{b_1} \leq 0.28$	at $\alpha = 0.05$	and	$-0.403 \leq \sqrt{b_1} \leq 0.403$	at $\alpha = 0.01$
$2.51 \leq b_2 \leq 3.57$	at $\alpha = 0.05$	and	$2.37 \leq b_2 \leq 3.98$	at $\alpha = 0.01$
$KS \leq 1.36/\sqrt{N}$	at $\alpha = 0.05$	and	$KS \leq 1.63/\sqrt{N}$	at $\alpha = 0.01$
$\chi^2(3) = 21.0$	at $\alpha = 0.05$	and	$\chi^2(3) = 26.2$	at $\alpha = 0.01$

collected at every 4010 million volume of trade based on the London Stock Exchange as an approximation since FT30 and FT100 are not real instruments for trading. The reason for 4010 million is a convenience to compare the result with those from Tables 5.3 and 5.4 since 4010 million is an average weekly trading volume during the period.

While this is a very crude approximation to market activity, the results are very encouraging. Table 5.6 shows strong support for the subordinated stochastic process hypothesis with a volume-normalization. Both indices are not significant under the normal distribution hypothesis at $\alpha = 0.05$. The total

Table 5.8 Price change at every equal amount of traded volume based on fortnightly averages

	B–J	$\sqrt{b_1}$	b_2	Klein's χ^2 (3)	K–S
FT30 Index	0.957	−0.096	2.485	0.314	0.045
FT100 Index	1.604	−0.130	2.337	3.988	0.063
Allied-Lyons	1.593	−0.321	3.300	5.509	0.073
Asda-MFI	0.615	−0.180	2.748	3.013	0.071
BICC	4.137	−0.567	3.132	4.699	0.080
BOC	3.396	−0.508	2.803	4.334	0.046
BTR	0.491	−0.161	2.774	1.228	0.050
Blue Circle	2.417	0.415	2.730	7.915	0.062
Boots	0.827	−0.124	2.553	4.799	0.074
British Airways	3.201	−0.472	3.341	4.548	0.090
British Gas	2.436	0.275	2.318	3.839	0.063
British Petrol	1.132	0.076	2.422	0.810	0.051
British Telecom	1.633	−0.317	2.664	7.829	0.087
Cadbury	2.276	0.188	3.759	5.967	0.117
Courtaulds	1.891	−0.171	2.307	2.885	0.060
Gen. Electric	2.177	0.367	3.383	2.172	0.054
Glaxo	1.090	−0.252	3.299	2.137	0.053
Grand Metro.	3.226	−0.491	3.231	4.227	0.051
GKN	1.304	−0.221	2.536	2.279	0.047
Guinness	8.824	−0.440	4.417	2.205	0.048
Hanson Trust	0.856	−0.194	2.655	2.842	0.055
Hawker Siddeley	8.218	−0.718	3.727	7.689	0.114
ICI	3.397	−0.409	2.365	6.110	0.077
Lucas	2.768	−0.467	3.016	2.885	0.069
M & S	15.47	0.817	4.488	6.764	0.078
Nat. West. Bank	52.34	0.527	6.926	5.602	0.077
P & O	4.036	0.002	4.128	0.704	0.070
Royal Ins.	0.200	−0.118	2.914	2.444	0.047
Tate & Lyle	4.549	−0.449	3.793	1.093	0.053
Thorn-EMI	0.343	0.087	3.279	0.482	0.059
Trusthouse	2.089	−0.347	2.580	0.412	0.090

$\chi^2(2) = 5.99$	at $\alpha = 0.05$	and	$\chi^2(2) = 9.21$	at $\alpha = 0.01$
$-0.23 \leq \sqrt{b_1} \leq 0.28$	at $\alpha = 0.05$	and	$-0.403 \leq \sqrt{b_1} \leq 0.403$	at $\alpha = 0.01$
$2.51 \leq b_2 \leq 3.57$	at $\alpha = 0.05$	and	$2.37 \leq b_2 \leq 3.98$	at $\alpha = 0.01$
$KS \leq 1.36/\sqrt{N}$	at $\alpha = 0.05$	and	$KS \leq 1.63/\sqrt{N}$	at $\alpha = 0.01$
$\chi^2(3) = 7.81$	at $\alpha = 0.05$	and	$\chi^2(3) = 11.3$	at $\alpha = 0.01$

trade-volume of the London Exchange was used for indices measured in millions since the indices are not traded, and the trade-volume for individual companies is measured in thousands. When applied to individual companies, the normality was also improved by a 20% increase in the numbers of companies that satisfy all tests (see Table 5.7).

Finally, relating to footnote 5, we attempted to improve our $K - S$ test by eliminating the mean and variance. This can be chosen by the following argument. The rates of return normalized by volume $(S_t - S_t - _1)/ V_t$ and

Table 5.9 $(S_2 - S_1)/V_2$, $(S_3 - S_2)/V_3$, $(S_4 - S_3)/V_4$, ... (daily price)

	B–J	$\sqrt{b_1}$	b_2	Klein's χ^2 (12)	K–S
FT30 Index	22.70	−0.191	3.758	46.68	0.034
Allied-Lyons	50.36	0.273	4.140	25.28	0.073
Asda-MFI	10.68	−0.028	3.579	73.07	0.072
BICC	226206	−4.566	87.19	133.3	0.212
BOC	35.60	−0.049	4.058	58.73	0.052
BTR	36.33	−0.238	3.962	43.34	0.064
Beecham	1.714	−0.129	2.807	25.99	0.057
Blue Circle	166.3	−0.232	5.249	56.53	0.070
Boots	12.37	0.113	3.584	20.42	0.063
British Airways	172.8	−0.084	5.334	60.98	0.067
British Gas	9.164	0.179	3.402	29.51	0.052
British Petrol	64.50	−0.052	4.426	36.15	0.045
British Telecom	0.070	0.011	3.042	49.68	0.054
Cadbury	11.40	0.124	3.548	47.49	0.046
Courtaulds	135.7	−0.084	5.067	51.94	0.053
Gen. Electric	15.40	−0.075	3.682	58.30	0.067
Glaxo	8.491	0.052	3.508	30.55	0.020
Grand Metro.	3.285	0.078	2.717	31.16	0.031
GKN	113.5	−0.104	4.886	63.84	0.050
Guinness	7.453	0.010	3.486	40.74	0.044
Hanson Trust	38.45	−0.169	4.051	59.29	0.081
Hawker Siddeley	1215.4	−0.055	9.206	72.26	0.079
ICI	10.60	−0.215	3.389	33.16	0.044
Lucas	1060.2	−0.041	8.797	116.2	0.070
M & S	3.929	−0.039	3.344	90.39	0.080
Nat. West. Bank	302.6	0.265	6.051	55.90	0.069
P & O	4.680	−0.099	3.331	46.10	0.038
Royal Ins.	45.25	−0.227	4.108	95.98	0.059
Tate & Lyle	210.5	−0.182	5.557	122.2	0.104
Thorn-EMI	11.44	−0.246	3.348	41.28	0.036
Trusthouse	275.0	0.422	5.829	21.27	0.051

$\chi^2(2) = 5.99$	at $\alpha = 0.05$	and	$\chi^2(2) = 9.21$	at $\alpha = 0.01$
$-0.23 \le \sqrt{b_1} \le 0.28$	at $\alpha = 0.05$	and	$-0.403 \le \sqrt{b_1} \le 0.403$	at $\alpha = 0.01$
$2.51 \le b_2 \le 3.57$	at $\alpha = 0.05$	and	$2.37 \le b_2 \le 3.98$	at $\alpha = 0.01$
$KS \le 1.36/\sqrt{N}$	at $\alpha = 0.05$	and	$KS \le 1.63/\sqrt{N}$	at $\alpha = 0.01$
$\chi^2(27) = 40.1$	at $\alpha = 0.05$	and	$\chi^2(27) = 47.0$	at $\alpha = 0.01$

$(S_{t+2} - S_{t+1})/\ V_{t+2} - (S_{t+1} - S_t)/\ V_{t+1}$ where S is the logarithmic price, were tested for the daily and weekly data. The latter one is motivated for the Kolmogorov–Smirnov test because it doesn't require specifying any parameter except the variance. Of course, since the variance is unknown, we have to estimate it and the discrepancy from the 'true' variable is quite minimal – see footnote 5. The reason of normalization by volume is that traded volume reflects the market activities, upon which the behaviour of prices depends. Then, the result is quite close to the normal distribution. Tables 5.7–5.10

Table 5.10 $(S_2 - S_1)/V_2$, $(S_3 - S_2)/V_3$, $(S_4 - S_3)/V_4$, ... (weekly price)

	B–J	$\sqrt{b_1}$	b_2	Klein's χ^2 (12)	K–S
FT30 Index	6.753	0.294	3.832	21.92	0.054
Allied-Lyons	13.44	0.298	4.309	11.11	0.061
Asda-MFI	2.147	−0.198	2.583	10.15	0.050
BICC	1.197	0.042	2.579	9.926	0.051
BOC	2.339	0.264	3.283	11.39	0.059
BTR	1.330	−0.134	2.636	8.070	0.044
Beecham	–	–	–	–	–
Blue Circle	10.74	0.459	3.899	9.616	0.050
Boots	1.000	0.047	2.619	14.58	0.045
British Airways	0.366	0.101	2.877	16.97	0.060
British Gas	35.00	0.294	5.245	16.28	0.055
British Petrol	0.855	0.151	2.800	9.522	0.046
British Telecom	0.205	−0.007	2.823	4.047	0.049
Cadbury	7.519	0.534	2.875	20.86	0.056
Courtaulds	0.128	−0.070	2.986	12.43	0.035
Gen. Electric	1.823	0.032	2.474	19.53	0.056
Glaxo	1.520	0.183	3.315	21.54	0.057
Grand Metro.	0.561	−0.108	2.801	8.944	0.039
GKN	1.633	−0.240	3.146	10.38	0.044
Guinness	1.322	0.222	3.077	6.755	0.029
Hanson Trust	1.935	−0.272	2.958	18.03	0.058
Hawker Siddeley	4.149	−0.396	3.108	15.97	0.068
ICI	0.378	0.040	3.228	6.476	0.030
Lucas	20.33	−0.163	4.738	29.72	0.057
M & S	1.290	0.170	2.713	7.605	0.053
Nat. West. Bank	12.33	0.456	4.033	17.23	0.063
P & O	2.634	−0.188	3.513	6.908	0.050
Royal Ins.	5.126	0.330	3.595	17.27	0.041
Tate & Lyle	2.550	0.211	3.462	19.52	0.062
Thorn-EMI	1.390	−0.190	2.737	20.31	0.061
Trusthouse	0.912	0.187	3.001	7.984	0.048

$\chi^2(2) = 5.99$	at $\alpha = 0.05$	and	$\chi^2(2) = 9.21$	at $\alpha = 0.01$
$-0.23 \leq \sqrt{b_1} \leq 0.28$	at $\alpha = 0.05$	and	$-0.403 \leq \sqrt{b_1} \leq 0.403$	at $\alpha = 0.01$
$2.51 \leq b_2 \leq 3.57$	at $\alpha = 0.05$	and	$2.37 \leq b_2 \leq 3.98$	at $\alpha = 0.01$
$KS \leq 1.36/\sqrt{N}$	at $\alpha = 0.05$	and	$KS \leq 1.63/\sqrt{N}$	at $\alpha = 0.01$
$\chi^2(12) = 21.0$	at $\alpha = 0.05$	and	$\chi^2(12) = 26.2$	at $\alpha = 0.01$

clearly show the improvement upon the normalization by volume, but the effect on the *K–S* test is minimal.

To summarize the results of our transformations, we shall use the Bera–Jarque statistic, which could be thought of as a quadratic loss function in skewness and kurtosis. For the 29 companies in the FT30, excluding Beecham because of merger within the data period, the average value of the Bera–Jarque for weekly data is 22.91 (Table 5.2), for the equal volume case it is 8.549 (Table 5.7) and for the volume weighted case it is 4.694 (Table 5.10). If we

Table 5.11 $(S_3 - S_2)/V_3$, $(S_2 - S_1)/V_3$, $(S_5 - S_4)/V_3$, $(S_4 - S_3)/V_4$ ... (daily price)

	B–J	$\sqrt{b_1}$	b_2	Klein's χ^2 (27)	K–S
FT30 Index	0.929	−0.095	2.849	21.17	0.027
Allied-Lyons	35.08	−0.125	4.472	22.59	0.056
Asda-MFI	10.68	−0.028	3.579	73.07	0.072
BICC	130828	−6.929	93.08	123.6	0.149
BOC	39.46	0.099	4.570	29.73	0.053
BTR	33.19	0.098	4.438	39.98	0.058
Beecham	0.458	0.038	3.224	2.118	0.041
Blue Circle	202.4	−0.125	6.576	39.48	0.051
Boots	34.50	0.183	4.434	36.13	0.052
British Airways	285.6	−0.307	7.214	32.00	0.043
British Gas	3.674	0.172	3.339	18.80	0.038
British Petrol	23.89	−0.223	4.148	60.07	0.050
British Telecom	3.364	−0.048	3.452	31.24	0.062
Cadbury	9.237	0.335	3.370	47.44	0.093
Courtaulds	144.1	0.016	6.024	38.94	0.059
Gen. Electric	5.820	−0.066	3.594	31.35	0.045
Glaxo	13.62	−0.051	3.924	26.65	0.060
Grand Metro.	0.014	0.012	2.982	33.34	0.031
GKN	105.8	0.649	5.244	94.64	0.091
Guinness	36.57	0.032	4.522	24.74	0.030
Hanson Trust	13.34	0.162	3.861	25.56	0.047
Hawker Siddeley	1211.9	0. 974	11.55	39.46	0.074
ICI	12.83	−0.450	8.345	84.28	0.096
Lucas	462.8	−0.450	8.345	84.28	0.096
M & S	8.824	0.135	3.698	33.56	0.043
Nat. West. Bank	319.8	0.174	7.493	55.78	0.102
P & O	3.829	−0.002	3.493	39.94	0.053
Royal Ins.	27.43	0.147	4.287	42.86	0.062
Tate & Lyle	47.97	−0.090	4.736	46.82	0.073
Thorn-EMI	2.172	0.108	3.302	26.47	0.058
Trusthouse	240.7	0.285	6.867	32.38	0.068

$\chi^2(2) = 5.99$	at $\alpha = 0.05$	and	$\chi^2(2) = 9.21$	at $\alpha = 0.01$
$-0.23 \leq \sqrt{b_1} \leq 0.28$	at $\alpha = 0.05$	and	$-0.403 \leq \sqrt{b_1} \leq 0.403$	at $\alpha = 0.01$
$2.51 \leq b_2 \leq 3.57$	at $\alpha = 0.05$	and	$2.37 \leq b_2 \leq 3.98$	at $\alpha = 0.01$
$KS \leq 1.36/\sqrt{N}$	at $\alpha = 0.05$	and	$KS \leq 1.63/\sqrt{N}$	at $\alpha = 0.01$
$\chi^2(27) = 40.1$	at $\alpha = 0.05$	and	$\chi^2(27) = 47.0$	at $\alpha = 0.01$

throw out the largest in each case and divide by 28, we get 9.86, 6.94 and 3.61 respectively. Further, when we used a volume amount equivalent to the average of two weeks' trade, only three out of 31 time series rejected the Bera–Jarque statistic (Table 5.8), which is also consistent with the fact that the nature of the return distribution becomes normal as the period length increases. We might hope that these adjustments to normalize each firm, indeed using our, admittedly crude, adjustment brings about a substantial improvement. For the FT30 the corresponding Bera–Jarque values are 30.28,

Table 5.12 $(S_3 - S_2)/V_3$, $(S_2 - S_1)/V_2$, $(S_5 - S_4)/V_5$, $(S_4 - S_3)/V_4$ ... (weekly price)

	B–J	$\sqrt{b_1}$	b_2	Klein's χ^2 (12)	K–S
FT30 Index	7.371	0.031	4.505	1.219	0.089
Allied-Lyons	12.69	−0.078	4.970	7.715	0.104
Asda-MFI	0.270	0.131	2.880	3.767	0.069
BICC	0.309	−0.090	2.750	2.123	0.083
BOC	0.437	0.180	3.073	2.160	0.070
BTR	3.806	−0.012	4.082	2.474	0.054
Beecham	–	–	–	–	–
Blue Circle	21.56	−0.305	5.502	9.549	0.101
Boots	0.873	−0.199	3.331	0.558	0.068
British Airways	2.590	−0.311	3.639	6.538	0.103
British Gas	72.68	1.140	7.143	4.219	0.134
British Petrol	1.164	0.286	2.823	1.105	0.087
British Telecom	0.100	−0.004	2.825	0.408	0.056
Cadbury	0.493	−0.148	2.747	0.729	0.054
Courtaulds	1.673	−0.109	2.316	5.131	0.096
Gen. Electric	0.137	0.010	2.796	3.237	0.071
Glaxo	0.380	−0.027	3.338	1.087	0.033
Grand Metro.	0.975	−0.250	3.224	4.534	0.073
GKN	0.323	−0.051	3.299	1.910	0.076
Guinness	0.268	−0.114	2.825	2.802	0.061
Hanson Trust	1.015	0.035	3.555	3.561	0.092
Hawker Siddeley	0.636	0.029	2.562	2.001	0.093
ICI	0.311	0.152	3.059	0.569	0.045
Lucas	1.716	0.031	3.724	4.377	0.131
M & S	1.136	0.240	3.346	2.381	0.072
Nat. West. Bank	8.706	0.676	3.922	7.570	0.062
P & O	4.441	−0.053	4.164	3.635	0.074
Royal Ins.	1.369	0.321	2.908	4.688	0.062
Tate & Lyle	1.961	−0.320	3.440	2.328	0.113
Thorn-EMI	2.735	−0.321	3.655	2.615	0.091
Trusthouse	0.563	−0.041	3.408	3.487	0.100

$\chi^2(2) = 5.99$	at $\alpha = 0.05$	and	$\chi^2(2) = 9.21$	at $\alpha = 0.01$
$-0.23 \leq \sqrt{b_1} \leq 0.28$	at $\alpha = 0.05$	and	$-0.403 \leq \sqrt{b_1} \leq 0.403$	at $\alpha = 0.01$
$2.51 \leq b_2 \leq 3.57$	at $\alpha = 0.05$	and	$2.37 \leq b_2 \leq 3.98$	at $\alpha = 0.01$
$KS \leq 1.36/\sqrt{N}$	at $\alpha = 0.05$	and	$KS \leq 1.63/\sqrt{N}$	at $\alpha = 0.01$
$\chi^2(12) = 21.0$	at $\alpha = 0.05$	and	$\chi^2(12) = 25.2$	at $\alpha = 0.01$

0.249, and 6.753. The improvement for the FT30 in the weekly equal volume case is quite remarkable. However, when we extended normality tests in the fortnightly data, the improvement is not so dramatic as in the weekly data. This is because a fortnight period is more normal and volume effects are averaged out in the fortnightly data (see Tables 5.3 and 5.8). We have not analyzed our fortnightly observations any further as they appear normal in the first case and the number of observations is only 78.

5.4 CONCLUSION

We tested the normality of speculative asset returns and indices in the London Stock Exchange. Our results are consistent with previous studies. The difference between the FT30 and the FT100 was one of our interests. If individual asset returns were log-normally (or log-stably) distributed, FT30 is expected to be better suited to a normal distribution. However, we found that there was little evidence to support this assumption. When we normalized by volume, FT30 performed better under the normal hypothesis. This gives some support for the subordinated stochastic process hypothesis rather than the stable Paretian distribution or the normal distribution hypotheses. Still, this finding is limited to the specific time-period and specific market, further theoretical work and methods are required.

The length of period has an importance for the nature of the return distribution. In empirical observations, the minimum satisfactory period is called for because the possibility of significant non-stationarity of the return distribution increases as the time period lengthens. The problem of stationarity occurs both intra- and inter-period. For example, in order to observe log-normality in monthly returns it is necessary that the process remain stationary not only during the individual months but also over the collection of months observed. If the underlying process were slowly changing, it might be that log-normality might be observed over some short sampling interval, but over some longer sampling log-normality might be rejected due to significant cumulative changes in the process. The question of the appropriate length of individual periods, and appropriate length of sampling interval, is empirical.

Finally one can interpret our results in two ways. In the literature that regards prices following a logarithmic Brownian motion, we have shown that the clock of the process is volume, not time. In the literature that is concerned with the distribution of share prices, we have shown that the conditional distribution of logarithmic price changes given volume is normally distributed. These two ideas are not mutually exclusive. We have not considered how to model volume. If we were to do so, we could, in principle, derive the marginal distribution of prices and examine its properties directly.

APPENDIX

We encounter several distributions, related to the normal distribution, which play important parts in the theory of statistics precisely because they are the forms taken by the sampling distributions of various statistics in samples from normal populations. The special position which the normal distribution holds, mainly by virtue of the Central Limit Theorem in one or other of its forms, is reflected in the positions of central importance occupied by these related distributions. Tests for normality can be divided into parametric and nonparametric tests depending on whether the alternative is given a parametric form or not.

The Kolmogorov–Smirnov test is a test of goodness of fit. Goodness-of-fit tests are based on a comparison of the hypothesized cumulative distribution function $F(x)$ with the empirical distribution function $F_n(x)$ obtained from a random sample of n observations. That is, it is concerned with the degree of agreement between the distribution of a set of sample values (observed scores) and some specified theoretical distribution. It determines whether the scores in the sample can reasonably be thought to have come from a population having the theoretical distribution. The test involves specifying the cumulative frequency distribution which would occur under the theoretical distribution. The point at which these two distributions, theoretical and observed, show the greatest divergence is determined. Reference to the sampling distribution indicates whether such a large divergence is likely on the basis of chance. Define $F_0(X) = a$ completely specified cumulative frequency distribution function, the theoretical cumulative distribution under H_0 and $S_N(X) =$ the observed cumulative frequency distribution of a random sample of N observations $= k/N$ where k is the number of observations equal to or less than X. Then, the test statistic is $D = \max_x |F_0(X) - S_N(X)|$. The distribution of D is not known for the case when certain parameters of the population have been estimated from the sample. However, Massey (1951) gives some evidence which indicates that if the $K - S$ test is applied in such cases (e.g. for testing goodness of fit to a normal distribution with mean and standard deviation estimated from the sample), the use of the table will lead to a conservative test. Empirically the $K - S$ test exhibits surprisingly poor power (see D'Agostino, 1971; D'Agostino and Pearson, 1973; Pearson, D'Agostino and Bowman, 1977).

The most widely used parametric tests for normality are those based on the skewness-kurtosis. The parametric alternative in these tests comes in the form of the Pearson family of densities. The goodness-of-fit tests are based on the sample second, third and fourth moment of the empirical distributions. These are given, respectively, by

$$\sqrt{b_1} = \frac{m_3}{m_2^{3/2}} \text{ and } b_2 = \frac{m_4}{m_2^2} \tag{A.1}$$

where

$$m_r = \sum_{i=1}^{n} (X_i - \overline{X})^r / n \text{ and } \overline{X} = \sum_{i=1}^{n} X_i.$$

The second moment is measure of spread or dispersion, the third moment is measure of skewness or asymmetry, and the fourth moment is measure of excess or kurtosis, which is the degree of flatness of a density near its centre. The normal distribution has the property that its third and fourth cumulants are both zero. Then, $\sqrt{b_1}$ is a good measure of non-normality against highly skewed and long-tailed distribution since all odd moments of a random variable about its mean are zero if the density function of random variable is symmetrical about the mean, provided such moments exist. And b_2 is sensitive to continuous, symmetric alternatives with heavy tails.

Under the null hypothesis of population normality, $\sqrt{b_1}$and b_2 are independent and their standardized normal equivalent deviates are approximately $X(\sqrt{b_1})$ and $X(b_2)$, where $X(\cdot)$ denotes a standardized normal distribution, hence $X^2(\sqrt{b_1}) + X^2(b_2)$ is asymptotically $\chi^2(2)$, for details see equation (A2). Bera and Jarque (1981) using the Pearson family as the parametric alternative derived the following skewness-kurtosis test as a Lagrange multiplier test. Let BJ be the Bera–Jarque statistic, then

$$BJ_n = [\frac{n}{6}\hat{b}_1 + \frac{n}{24}(\hat{b}_2 - 3)^2 - \chi^2(2) \qquad (A.2)$$

where

$$\sqrt{\hat{b}_1} = [\frac{1}{n}\sum_{t=1}^{n}\hat{u}_t^3) \Big/ (\frac{1}{n}\sum_{t=1}^{n}\hat{u}_t^2)^{\frac{3}{2}}]$$

$$\hat{b}_2 = [\frac{1}{n}\sum_{t=1}^{n}\hat{u}_t^4) \Big/ (\frac{1}{n}\sum_{t=1}^{n}\hat{u}_t^2)^2]$$

where $\hat{u}_t$ is typically a regression residual in our case $\hat{u}_t = r_t - \bar{r} = r$. Notice that equation (A2) is the same as (A1) but that (A2) allows one to consider residuals from linear regressions with sets of regression variables other than just a constant.Financial support from the Newton Trust and Inquire (U.K.) is gratefully acknowledged. The comments from David Damant (Paribas Asset Management) have been very helpful. This paper has been produced as a discussion paper for the Autumn 1991 Inquire Conference. Some of the contents are still rather preliminary.

ACKNOWLEDGEMENTS

Financial support from the Newton Trust and Inquire (UK) is gratefully acknowledged. The comments from David Damant (Paribas Asset Management) have been very helpful. This chapter was produced as a discussion paper

for the Autumn 1991 Inquire Conference. Some of the contents are still rather preliminary.

REFERENCES

Affleck-Graves, J. and McDonald, W. (1989) 'Nonnormalities and Tests of Asset Pricing Theories', *Journal of Finance*, **XLIV**, 889–908.

Alexander, S.S. (1961) 'Price Movements in Speculative Markets: Trends or Random Walks', *Industrial Management Review*, **2**, 7–26.

Bachelier, L. (1990) *Theory of Speculation*. Reprinted in *The Random Character of Stock Market Prices* (1964), edited by P.H. Cootner.

Balvers, R., Cosimano, T. and McDonald, B. (1990) 'Predicting Stock Returns in an Efficient Market', *Journal of Finance*, **XLV**, 1109–1128.

Bera, A.K. and Jarque, C.M. (1981) 'An Efficient Large-Sample Test for Normality of Observations and Regression Residuals', Australian National University Working Papers in Econometrics No. 40, Canberra.

Bera, A.K., Jarque, C.M. and Lee, L.-F. (1984) 'Testing The Normality Assumption in Limited Dependent Variable Models', *International Economic Review*, **25**, 563–578.

Clark, P.K. (1973) 'A Subordinate Stochastic Process Model with Finite Variance for Speculative Prices', *Econometrica*, **41**, 135–155.

D'Agostino, R.B. (1971) 'An Omnibus Test of Normality for Moderate and Large Size Samples', *Biometrika*, **58**, 341–348.

D'Agostino, R.B. and Pearson, E.S. (1973) 'Tests for Departure From Normality: Empirical Results for the Distribution of b_2 and $\sqrt{b_1}$', *Biometrika*, **60**, 613–622.

Epps, T.W. and Epps, R.E. (1976) 'The Stochastic Dependence of Security Price Changes and Transaction Volumes: Implications for the Mixture-of-Distribution Hypothesis', *Econometrica*, **44**, 305–321.

Fama, E.F. (1963) 'Mandelbrot and the Stable Paretian Hypothesis', *Journal of Business*, **36**, 420–429.

Fama, E.F. (1965) 'The Behavior of Stock Market Prices', *Journal of Business*, **38**, 34–105.

Fielitz, B.D. and Smith, E.W. (1972) 'Asymmetric Stable Distributions of Stock Price Changes', *Journal of the American Statistical Association*, **67**, 813–814.

Gastwirth, J.L. and Owens, M.E.B. (1977) 'On Classical Tests of Normality', *Biometrika*, **64**, 135–139.

Harris, L. and Gurel, E. (1986) 'Price and Volume Effects Associated with Changes in the S&P 500 List: New Evidence for the Existence of Price Pressures', *Journal of Finance*, **XLI**, 815–829.

Karpoff, J.M. (1987) 'The Relation Between Price Changes and Trading Volume: A Survey', *Journal of Financial and Quantitative Analysis*, **22**, 109–126.

Kendall, M.G. (1953) 'The Analysis of Economic Time-Series-Part I: Prices', *Journal of the Royal Statistical Society*, Series A, **CXVI**, 11–25.

Klein, L. (1974), *Textbook of Econometrics, Prentice-Hall.*

Kon, S.J. (1984) 'Models of Stock Returns – A Comparison', *Journal of Finance*, **39**, 147–165.

Leitch, R.A. and Paulson, A.S. (1975) 'Estimation of Stable Law Parameters: Stock Price Behavior Application', *Journal of the American Statistical Association*, **70**, 690–697.

Luskin, D.L. (1987) *Index Options & Futures*, John Wiley, New York.

Mandelbrot, B. (1963) 'The Variation of Certain Speculative Prices', *Journal of Business*, **36**, 394–419.

Massey, F.J. Jr. (1951) 'The Kolmogorov-Smirnov Test for Goodness of Fit', J*ournal of the American Statistical Association*, **46**, 68–78.

Osborne, M.F.M. (1959) 'Brownian Motion in the Stock Market', *Operations Research*, **7**, 145–173.

Pearson, E.S., D'Agostino, R.B. and Bowman, K.O. (1977) 'Tests for Departure from Normality: Comparison of Powers', *Biometrika*, **64**, 231–246.

Saniga, E.M. and Miles, J.A. (1979) 'Power of Some Standard Goodness-of-Fit Tests of Normality Against Asymmetric Stable Alternatives', *Journal of the American Statistical Association*, **74**, 861–865.

Shapiro, S.S. and Wilk, M.B. (1965) 'An Analysis of Variance Test for Normality (Complete Samples)', *Biometrika*, **52**, 591–611.

Shapiro, S.S., Wilk, M.B. and Chen, J. (1968) 'A Comparative Study of Various Tests for Normality', *Journal of the American Statistical Association*, **63**, 1343–1372.

Spanos, A. (1986) *Statistical Foundations of Econometric Modelling*, Cambridge University Press, Cambridge.

Tauchen, G.E. and Pitts, M. (1983) 'The Price Variability-Volume Relationship on Speculative Markets', *Econometrica*, **51**, 485–505.

Upton, D.E. and Shannon, D.S. (1979) 'The Stable Paretian Distribution, Subordinated Stochastic Processes, and Asymptotic Log-normality: An Empirical Investigation', *Journal of Finance*, **34**, 1031–1039.