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The interrelations between moiré patterns, contour fringes, optical surfaces and their sum and difference effects

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Abstract. A general relationship between potential functions and moiré patterns can be realized in a number of forms. The potential function can be worked as an optical surface and combined with other potential functions similarly worked to give the solution to a complex potential problem. Contour fringes can be produced from the optical surfaces or generated on a computer graph plotter directly. When superimposed these contour lines generate moiré fringes which are also solutions to potential function problems but with an essential ambiguity of sign. Two circularly symmetrical potential wells have been contoured and moiré fringe solutions to simple problems are given.

1. Introduction

In an earlier paper a method of calculating moiré patterns by using spatial frequency vectors was described [1]. It was also shown that the spatial frequency vectors can generally but not invariably be represented as the gradients of a potential surface. There are some cases, such as those where the potential is not single-valued, where special limits must be placed on the field. This is particularly the case for a set of radial lines where a cut must be imposed, e.g. at $\theta = 0$.

It is of importance and of optical interest to note that we can construct an optical element, e.g. a plano-aspheric surface, such that its thickness at any point represents the potential function. If we now photograph contour fringes between the aspheric surface and a reference plane, this gives us a representation of the corresponding generating grid.

There are now two closely related optical operations we may perform. We may take two plano-aspheric elements and place them in contact, with or without lateral displacements or twists. The resulting optical component will produce at any point a deviation which is the vector sum of the deviations in the two elements at the given point. Alternatively, we may use the corresponding grids to produce a moiré pattern when it will be found that the moiré pattern produces a diffractive deviation of an incident ray equivalent to the refractive deviation of the compound optical system.

Put another way, we can regard the grids as holograms of the corresponding optical elements and the moiré pattern as arising from the combination of two holograms.

In a very simple case we can regard two spherical surfaces—a plano-convex and a plano-concave lens—as particular cases of aspheric elements. The contour fringes of either will form a zone plate to a close approximation. Two zone plates placed with their centres laterally displaced [2] give rise to linear fringes equivalent to a thin prism. The corresponding lenses also give rise to a thin prism [3] if combined with their centres laterally displaced.

It will be noted that the grids have an essential ambiguity when used as diffracting elements, since an incident wave gives rise to two diffracted waves, one on each side of the incident direction. This is analogous to the essential ambiguity in holography [4–7] and arises from the fact that stationary contour fringes do not distinguish the hills from the valleys. In a Twyman–Green interferometer it is necessary to touch one of the elements to ascertain which way the fringes move.

If it is desired to produce an unambiguous optical result, plano-aspheric elements must be used rather than grids.

Another factor of these interlocking relationships is that a grid may be regarded as defining the contour fringes of *either* a convex *or* a concave surface. Grids will, in general, give rise to two sets of moiré fringes, one a sum set and one a difference set [1]. Thus two overlapping zone plates are equivalent either to two lenses of opposite sign—giving a prism—or two lenses of the same sign, giving a lens of twice the power centred mid-way between the other two centres [1, 8].

2. Lens or zone plates of variable power

Lohmann and Paris [9] have described a number of ways of generating grids which give a zone-plate moiré pattern of variable power, the power being a function of the relative positioning of the grids. Of these systems, by far the most important is the cubic function $z=x^3+y^3$ or its equivalent form $z=x(x^2+3y^2)$ obtained by an axial rotation and change of scale. A variable focus zone plate is obtained by displacing two copies of the former along the line $x=y$ or two copies of the latter along the x azis. This system occurred independently to one of us (L.C.G.R.) since two second-order functions (zone plates) give a variable linear output (prism) and it was argued that therefore two cubics would give a variable second-order output (zone plate). We suggested the use of this cubic to Burch and Williams, who are now developing an improved version for application to large-scale alignment [10].

Alvarez [11] has constructed the equivalent optical system using two plano-aspheric elements. He has also shown that a lens with a variable amount of astigmatism can be produced by displacing the elements oblique to the line producing circular symmetry.

3. Use of moiré patterns to solve potential problems

Potential field problems can in principle be solved by combining a number of plano-aspheric elements in an optical system. The construction of the elements can be difficult, and the resulting element is inflexible. It is relatively easy to generate the grids appropriate to the various potentials of the input and obtain sum and difference effects. It is also relatively easy to scale them in a photographic enlarger to correspond, e.g., to charges of different magnitude. Grids can be computer-generated [9] but in the special case of circular symmetry they can be cut on a lathe [12].

We have constructed circular grid patterns corresponding to a point charge [$z \propto (1/r)$] and a line charge ($z \propto \log r$). There is a limit to the extent to which a lathe will cut a fine line, and to avoid a big gap in the centre these systems have been built up in two stages. The outer rings are photographed direct from a master cut in a lathe, the black region being in hollows and the white regions (corresponding to the naked bare materials) being exposed by a skimming cut over the painted surface [12]. The inner rings are produced by a photographic print from another master originally cut on a much larger scale. In the case of the log law the same master does for each part, a reduced photo of the master being stuck in the middle.

Figures 1 (*a*) and (*b*) give the circular masters appropriate to a point and a line change respectively. Figure 2 (*a*) shows the equipotentials due to two equal

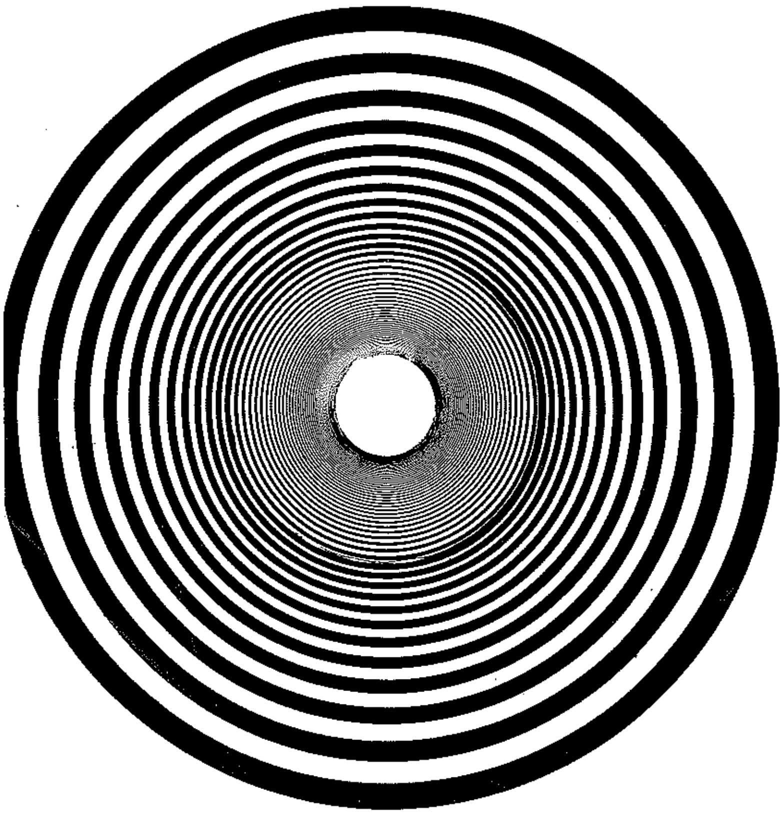
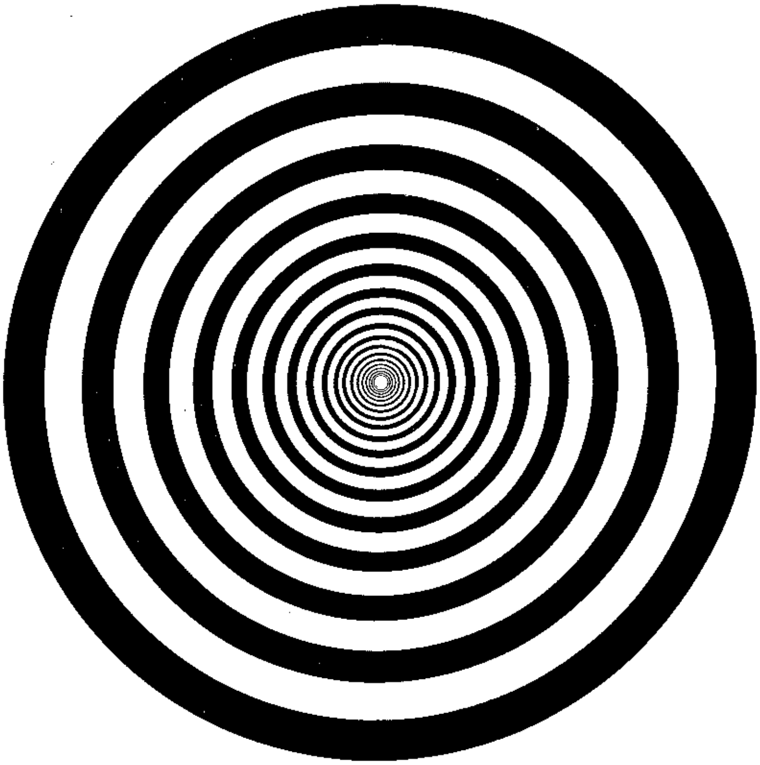


Figure 1 (*a*)

(*continued overleaf*)

Figure 1 (*continued*)



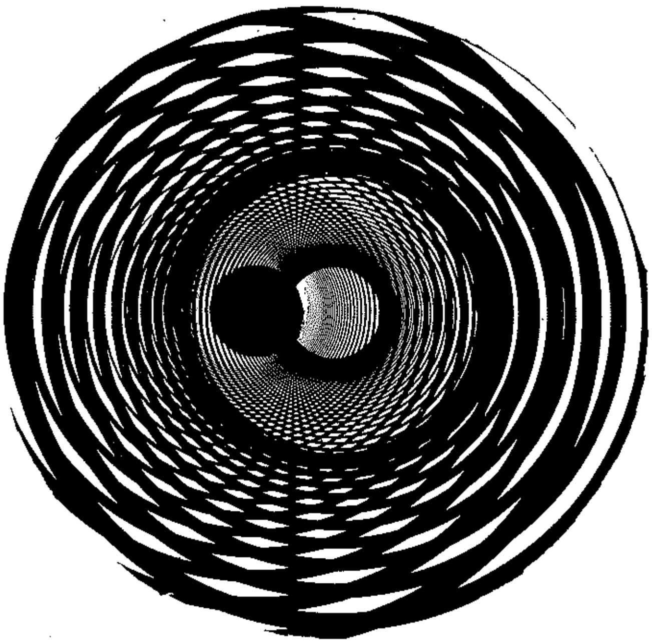
(*b*)

Figure 1. Contoured potential wells for (*a*) point charge, (*b*) line charge.

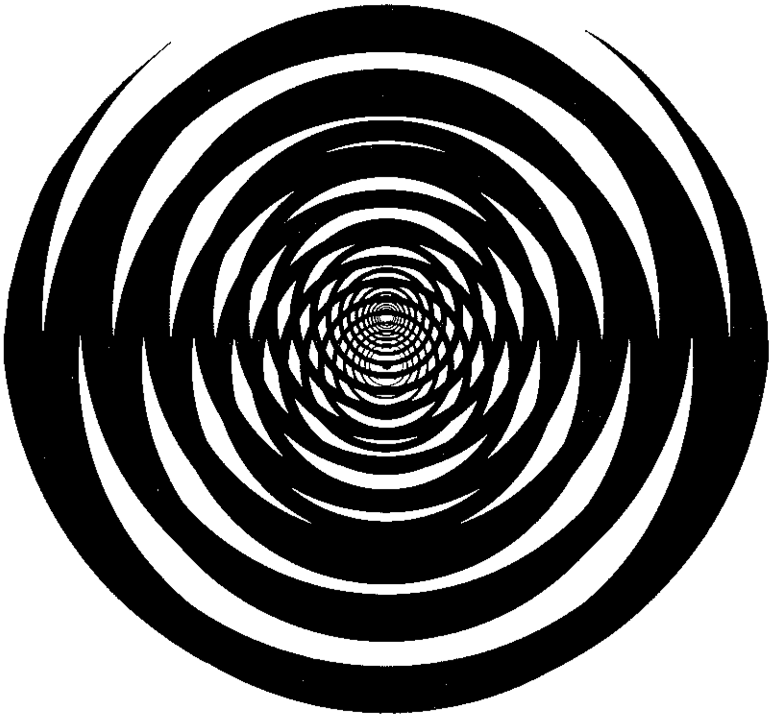
and opposite point charges and figure 2 (*b*) the corresponding curves for line changes. The moiré patterns map the equipotentials and, in order to bring out the straight line zero potential as a black line, a negative image of figure 1 (*a*) or (*b*) is used together with the corresponding positive image [13].

Figure 3 (*a*) shows the effect of superimposing two point-charge grids with their centres well separated. In this case a sum moiré pattern is generated showing the neutral point half-way between two equal charges of the same sign. Figure 3 (*b*) shows the effect of superimposing three grids. Neutral points due to charges taken in pairs are readily seen and the neutral point at the centre of the triangle can just be discerned. In principle a large number of grids could be superimposed but the moiré patterns become progressively fainter.

It is possible to study the effect of two unequal charges (figure 3 (*c*)) by reproducing the grids on different scales. Figure 3 (*d*) shows point and line charges with a closed-loop equipotential round the neutral point. Should it be necessary to find an exact solution to a multichange potential problem this can always be done uniquely if the corresponding plano-aspheric elements can be manufactured. There is now no ambiguity in the system and the overall optical system gives the equipotentials without confusion.

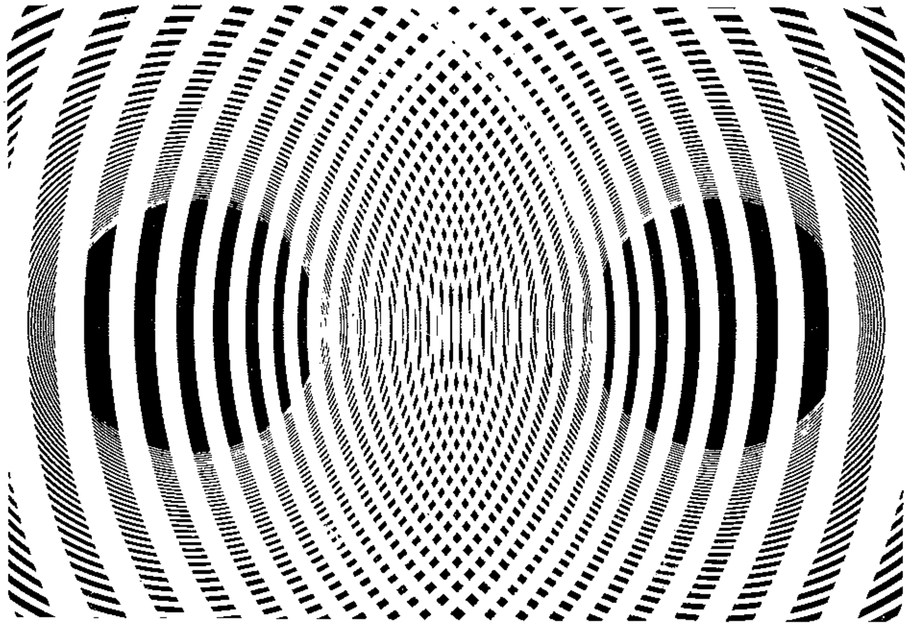


(*a*)

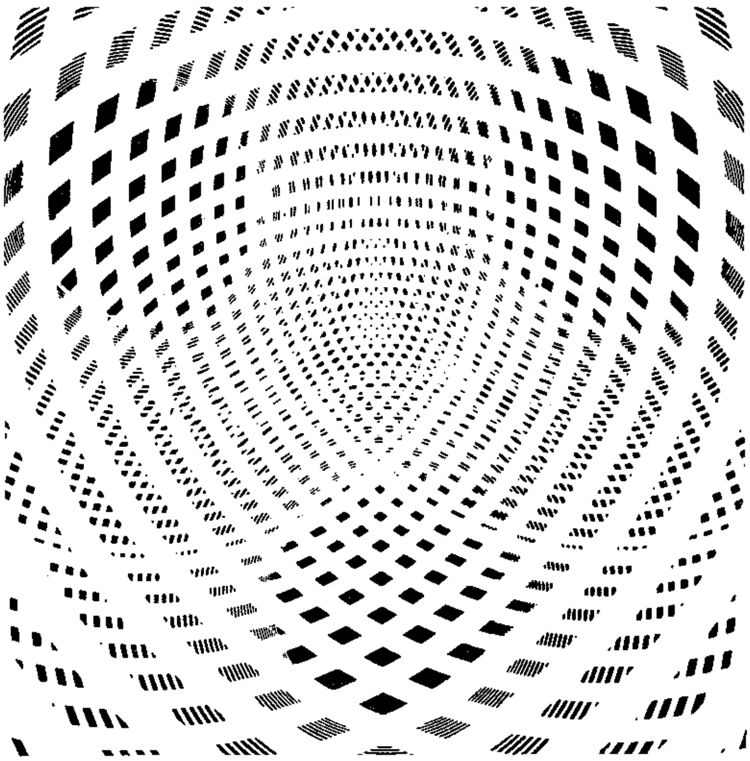


(*b*)

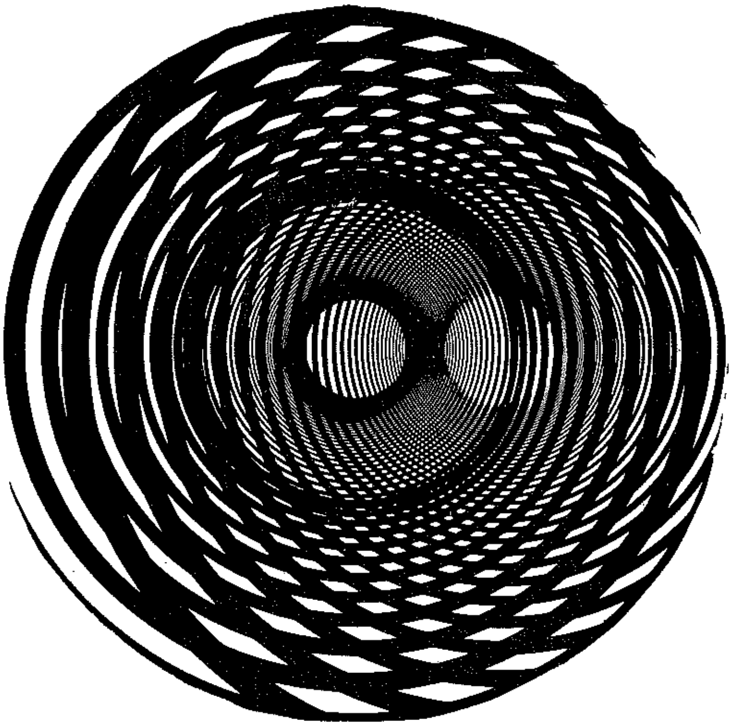
Figure 2. Equipotentials for equal and opposite charges; (*a*) point charges, (*b*) line charges.



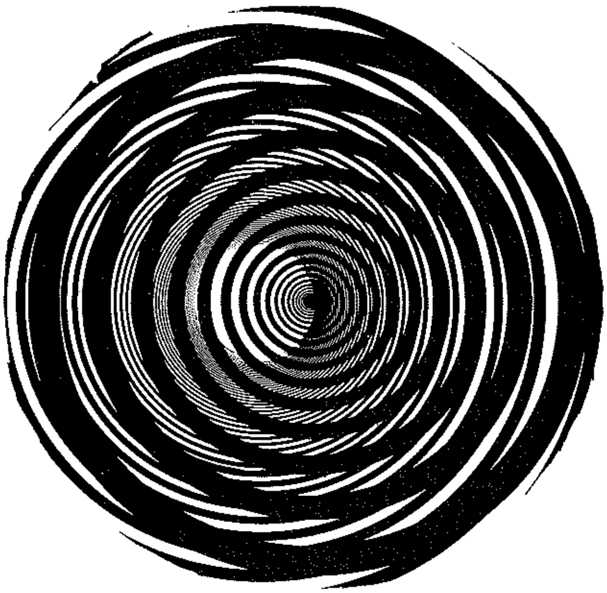
(a)



(b)



(*c*)



(*d*)

Figure 3. Equipotentials for (*a*) two equal and similar point charges, showing neutral point, (*b*) three equal and similar point charges showing triangular contours round neutral point, (*c*) two unequal and opposite point charges, (*d*) two unequal and opposite charges, one a point charge and the other a line charge. A closed equipotential contour can just be seen round the neutral point on one side.

Acknowledgments

We are greatly indebted to Mr. A. Mackenzie for his skill and patience in cutting circles on a lathe and for his accuracy in marrying the two stage systems to produce the masters of figure 1.

On peut réaliser de plusieurs façons une relation générale entre fonctions potentielles et figures de Moiré. La fonction potentielle peut être travaillée comme une surface optique et combinée avec d'autres fonctions potentielles travaillées de façon analogue afin de donner la solution d'un problème de potentiel complexe. Des franges de contour peuvent être produites à partir des surfaces optiques ou engendrées directement sur un traceur de courbes associé à une calculatrice. Lorsqu'elles sont superposées, ces lignes de contour engendrent des franges de Moiré, qui sont aussi des solutions de problèmes de fonction potentielle, mais avec une ambiguité de signe essentielle. On a formé les contours de deux puits de potentiel circulairement symétriques et l'on présente des solutions, à l'aide de franges de Moiré, pour des problèmes simples.

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