PERPETUAL DEFAULTABLE CALLABLE CONVERTIBLE BONDS

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Overview

THE NO-CALLING CASE

- **The model and the problem**
- **The solution**

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- **Block conversion of last m_0 bonds**
- **The case $K < K_c$**
- **The case $K > K_c$**

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(Ingersoll's paradox: bond issue should be called when bond value reaches the call price, yet in practice the bond value may go 40-80% above before calling!)

Model assumptions

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$$dV_t = V_t \left[\, \sigma dW_t + (r - \delta)dt \,\right] \tag{1}$$

for positive constants r, the riskless rate of interest, and δ, the rate of disbursement of value to shareholders *and bondholders*

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- Coupon payments are tax-deductible, tax rate is τ.

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- *What is the bondholders' optimal conversion behaviour?*
- *What is the optimal timing of default for the shareholders?*
- *What is the value of the convertible bond and of the share?*

Properties of the solution

The values of share and bond must be functions of m_t and V_t alone, where m_t is number of live convertibles at time t:

$$S_t = S(m_t, V_t), \quad B_t = B(m_t, V_t).$$

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In continuation region $\mathcal{C}$ where neither default nor conversion occurs,

$$\mathcal{L}B + \rho = 0, \quad \mathcal{L}S + \frac{\delta V - m\rho'}{n - m} = 0,$$

($\mathcal{L} \equiv \frac{1}{2}\sigma^2 V^2 \frac{\partial^2}{\partial V^2} + (r - \delta)V \frac{\partial}{\partial V} - r$, $\quad \rho' \equiv \rho(1 - \tau)$) because

$$Z_t = e^{-rt} B_t + \int_0^t \rho e^{-rs}\, ds, \qquad X_t = e^{-rt} S_t + \int_0^t \frac{\delta V_s - \rho' m_s}{n - m_s} e^{-rs}\, ds$$

are both martingales.

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Solution characterised by $\xi : [0, n] \to \mathbb{R}^+$ (the default boundary) and decreasing $\eta : [0, n] \to \mathbb{R}^+$ (the conversion boundary) and the rule:

convert to keep $V_t \leq \eta(m_t)$, default when $V_t < \xi(m_t)$.

Form of the solution

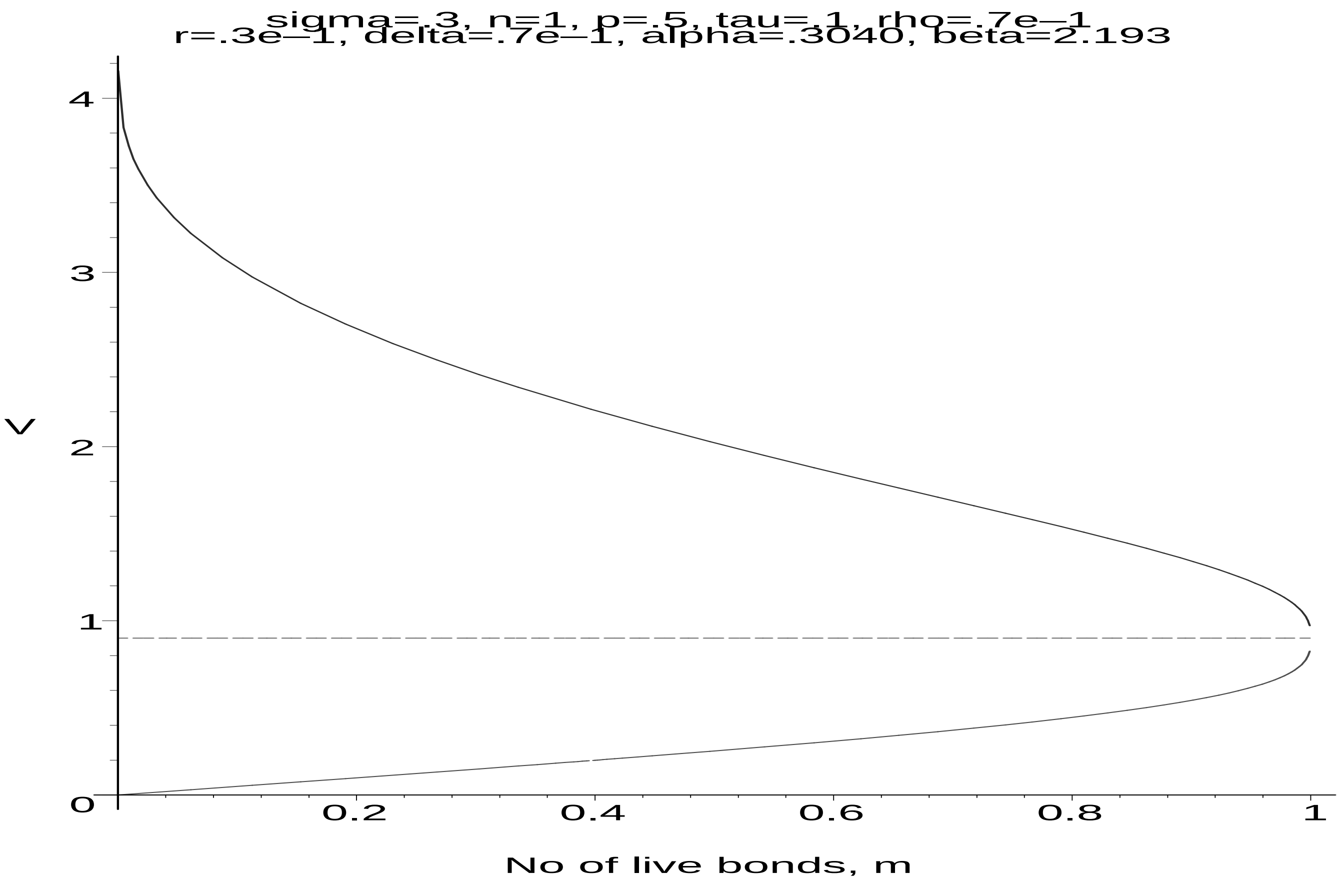
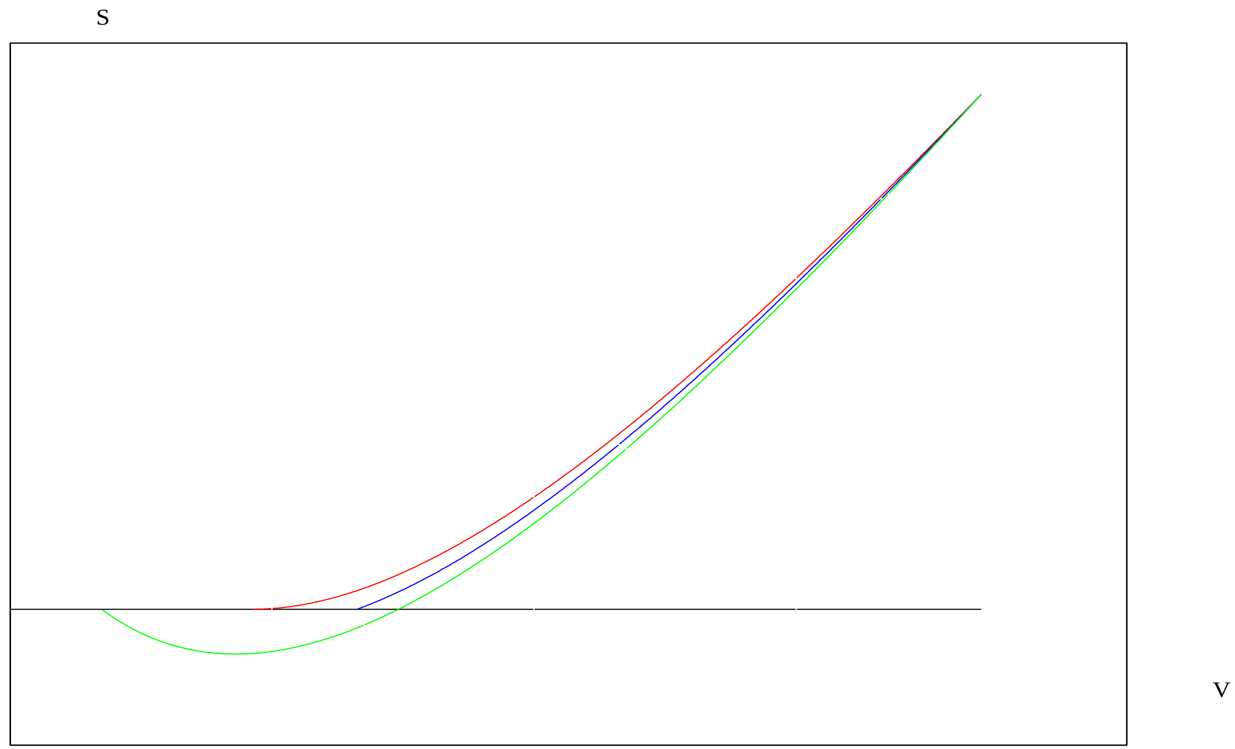


Figure 0: The conversion and bankruptcy regions for a typical convertible bond problem. Conversion occurs continuously along the upper boundary η, and bankruptcy is declared when the lower boundary ξ is reached.

Finding the solution

We have the important principle of local optimality: considering a single m-slice,



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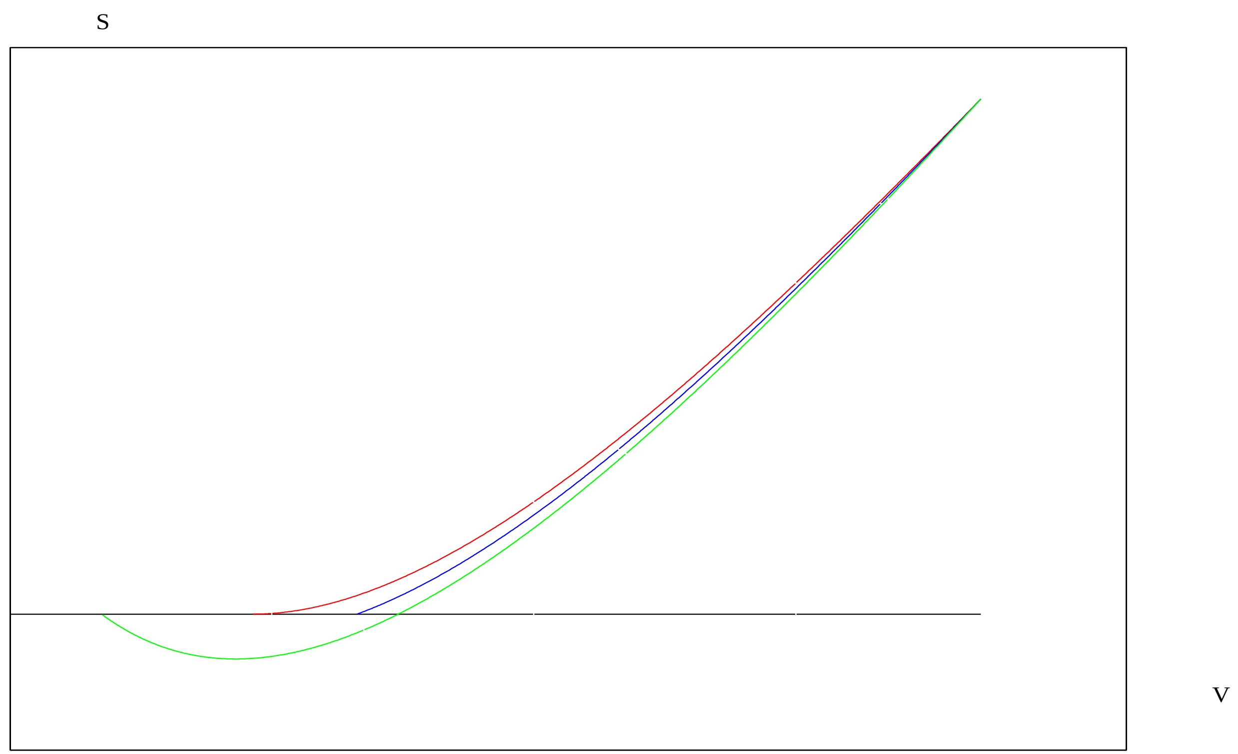

we deduce that $S_V = 0$ at default.

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\begin{aligned}
S(m,\xi(m)) &= 0 = S_V(m,\xi(m)) \\
Y(m,\eta(m)) &= 0 = Y_V(m,\xi(m)) \\
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lead to

$$S(m,V) = \frac{m\rho'\psi_0(V/\xi(m)) - rV\psi_1(V/\xi(m))}{r(\alpha+\beta)(n-m)},$$

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where $\beta > 1$ and $-\alpha < 0$ solve $\frac{1}{2}\sigma^2 x(x-1) + (r-\delta)x - r = 0$, and

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are strictly convex, nonegative, vanishing at 1.

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Hence we get $\eta(m)$ as a function of m and $\xi(m)$, and

a differential equation for $\xi(m)$.

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Best to work with independent variable

$$\theta = \xi(m)/\eta(m) \quad \in [0, 1]$$

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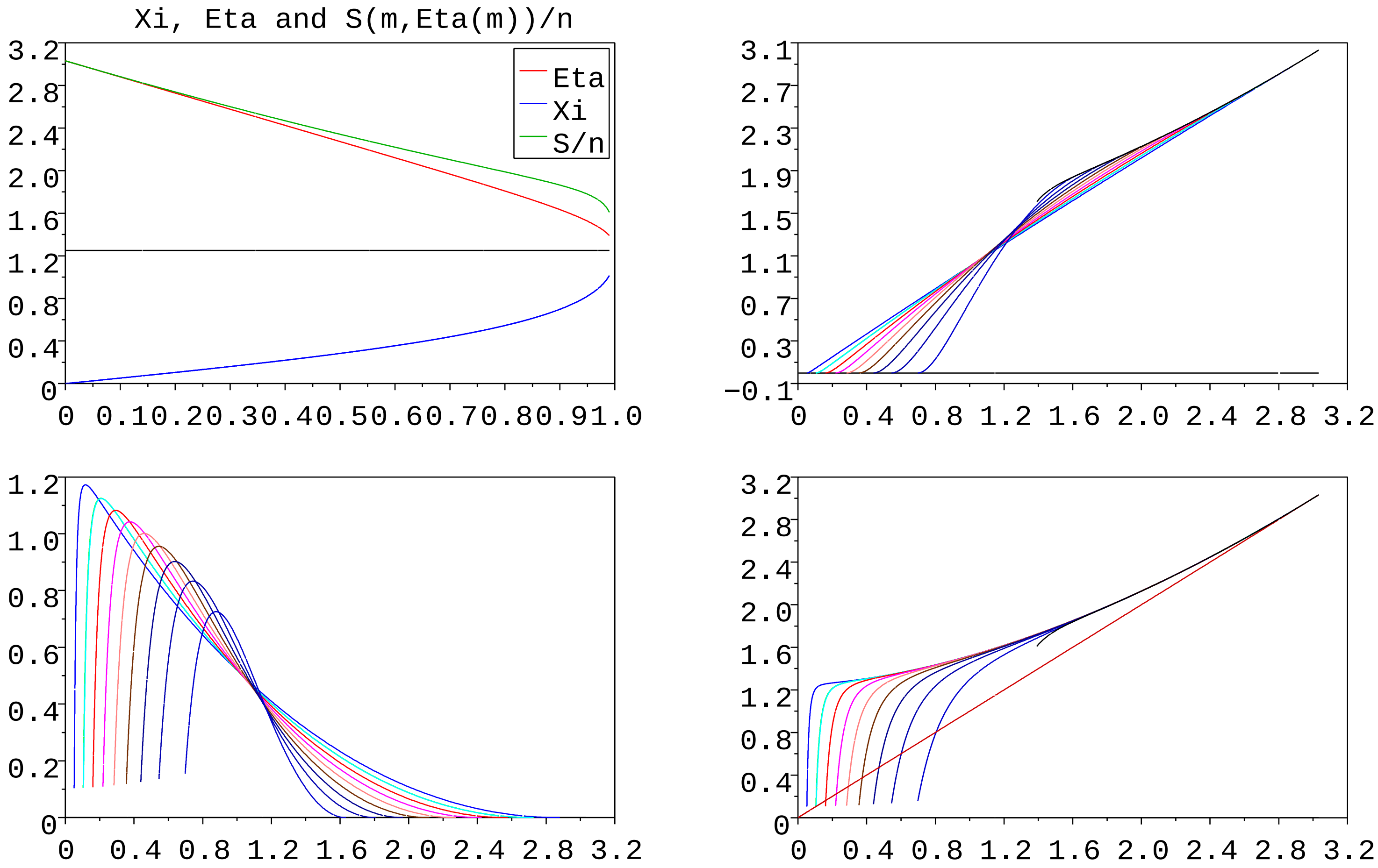
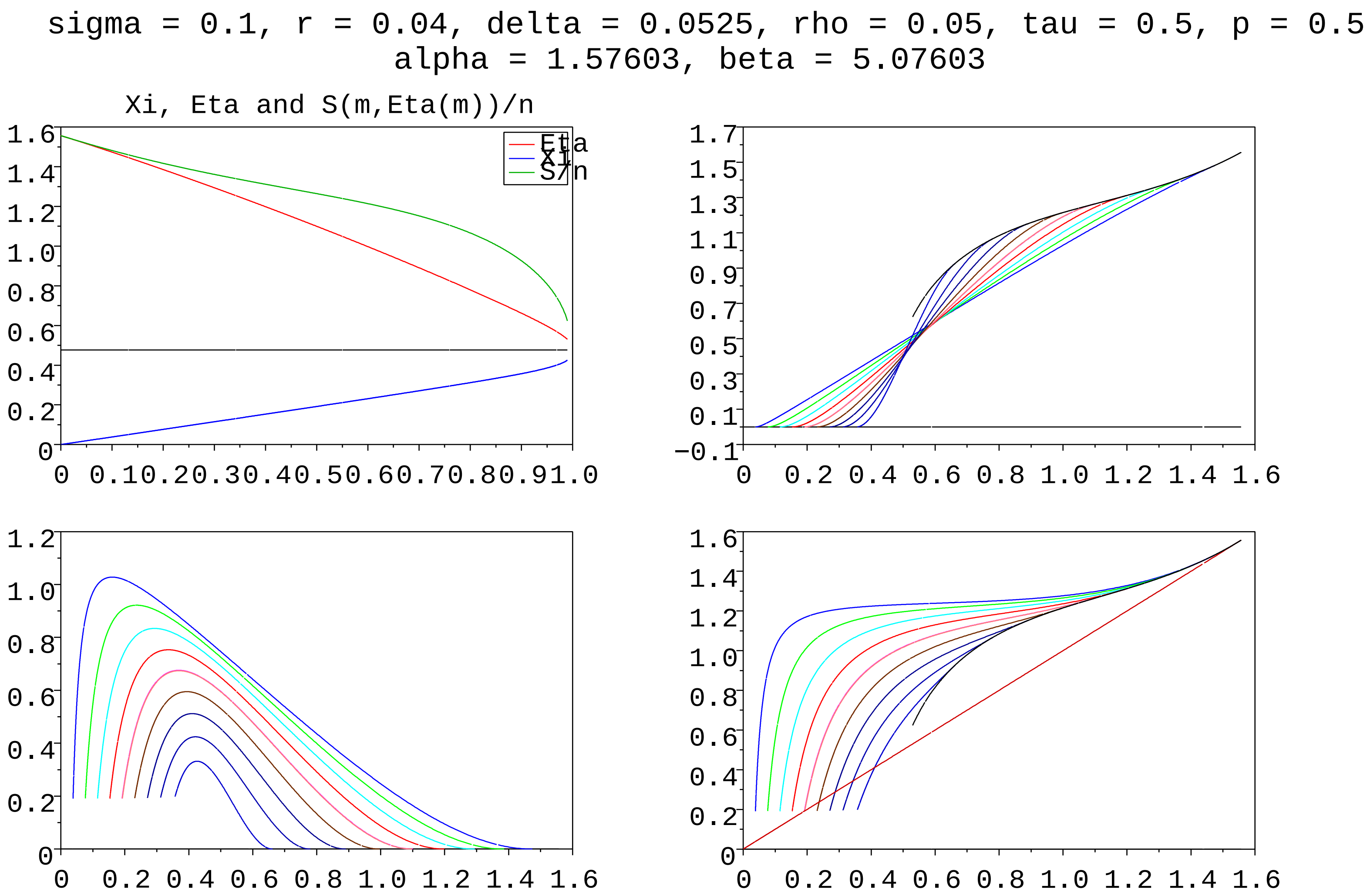
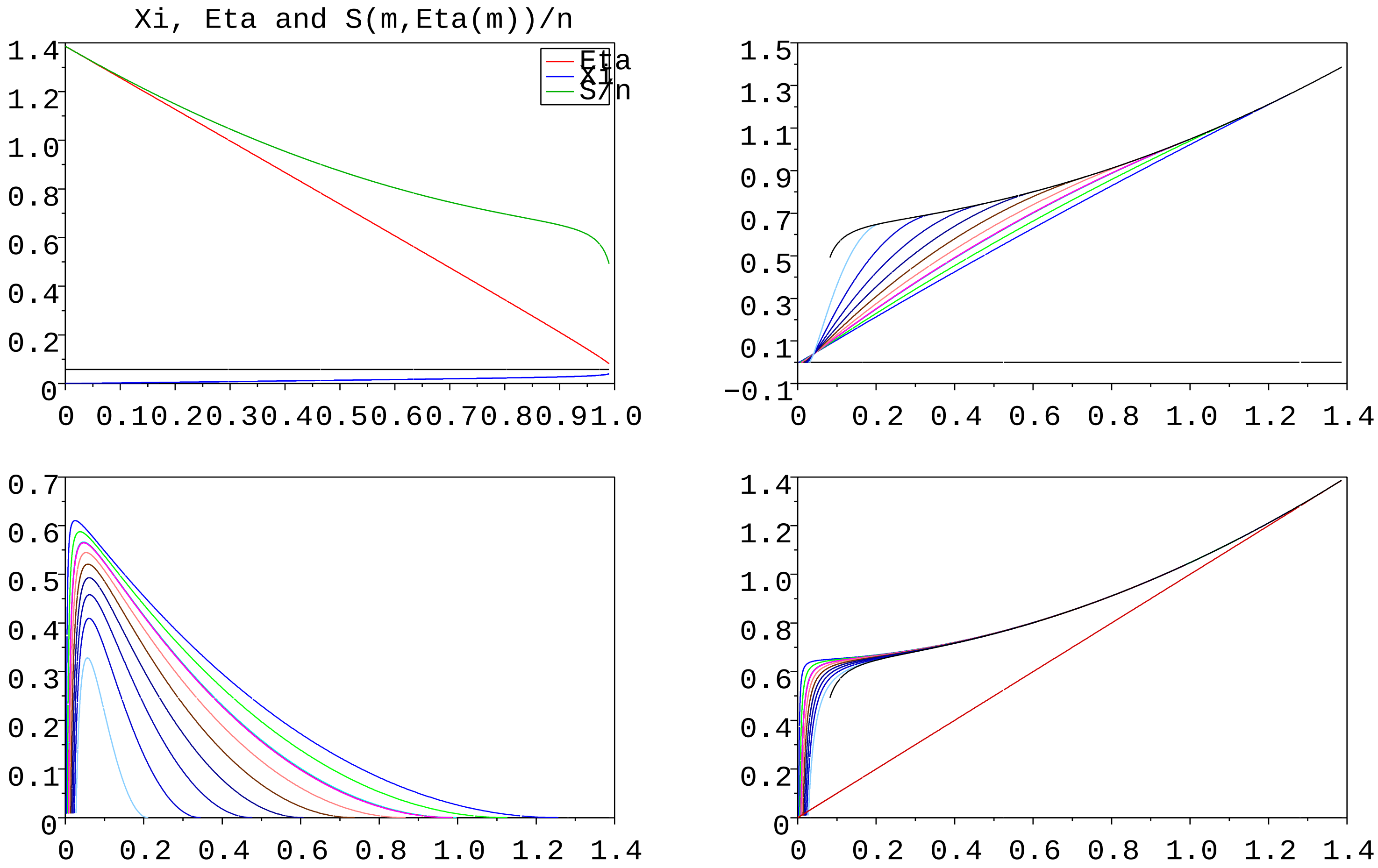
- Excursion-theoretic approach?

sigma = 0.1, r = 0.04, delta = 0.02, rho = 0.05, tau = 0.5, p = 0.2
alpha = 4.70156, beta = 1.70156




sigma = 0.1, r = 0.04, delta = 0.0525, rho = 0.05, tau = 0.5, p = 0.5
alpha = 1.57603, beta = 5.07603
Xi, Eta and S(m,Eta(m))/n
Eta
Xi
S/n

sigma = 1, r = 1.539, delta = 1.1627, rho = 1, tau = 0.933, p = 0.3076
alpha = 1.63508, beta = 1.88248



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where

$$S_0(V) = \frac{V}{n^2} - \frac{\rho(\beta\tau+1-\tau)}{nr(\beta-1)}\left(\frac{V}{\eta_0}\right)^{\beta} - \frac{\rho(1-\tau)}{nr}.$$

has unique root nK_c in $(0,\eta_0)$;

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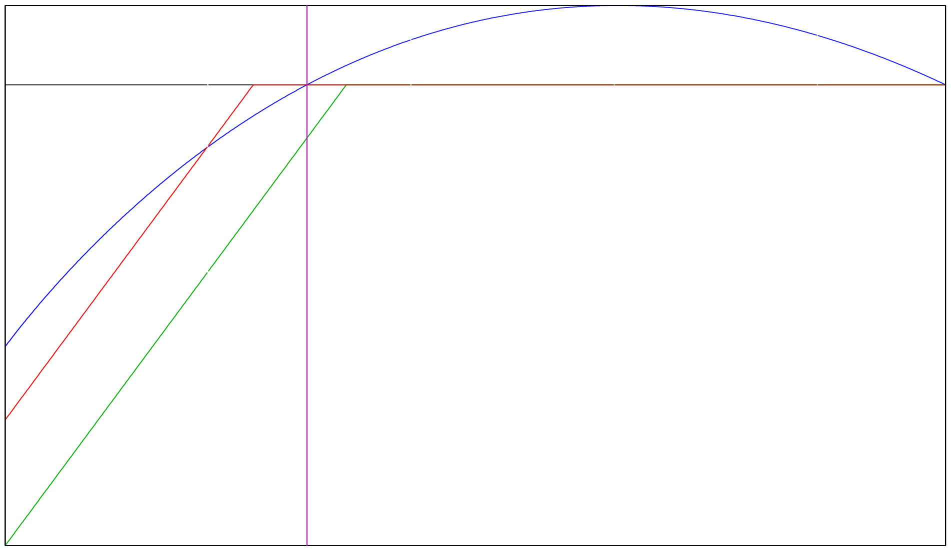

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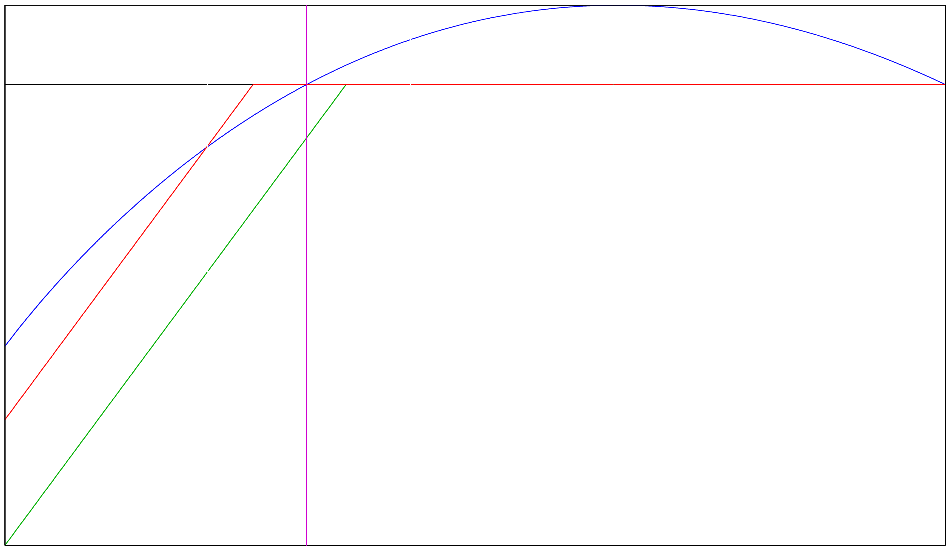

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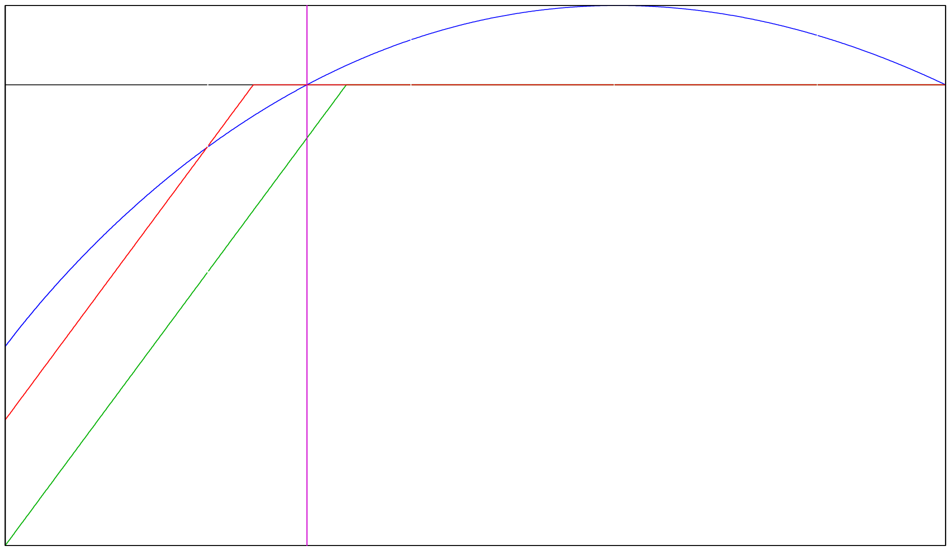
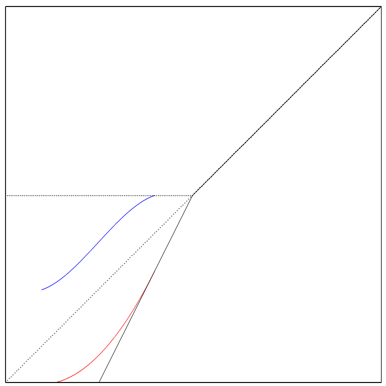


- If $K > K_c$, then no-calling solution holds for small enough m;
- If $K < K_c$, then no-calling solution gets changed immediately

Last m_0 bonds exercised at once; is it

Case 1



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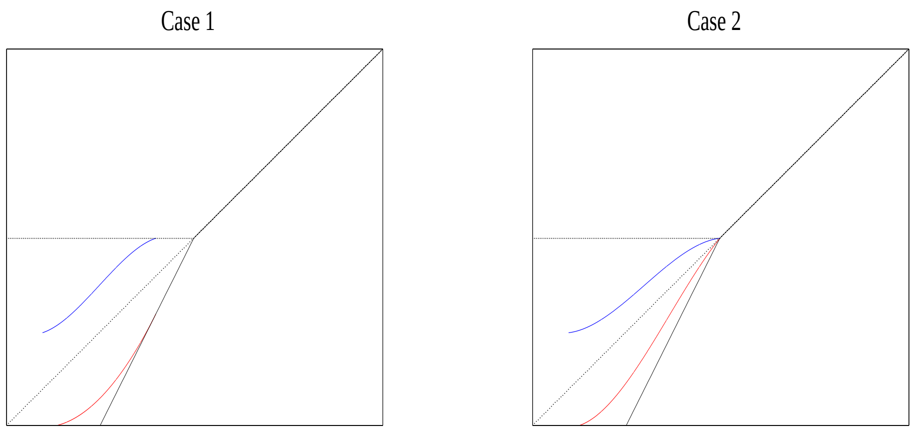

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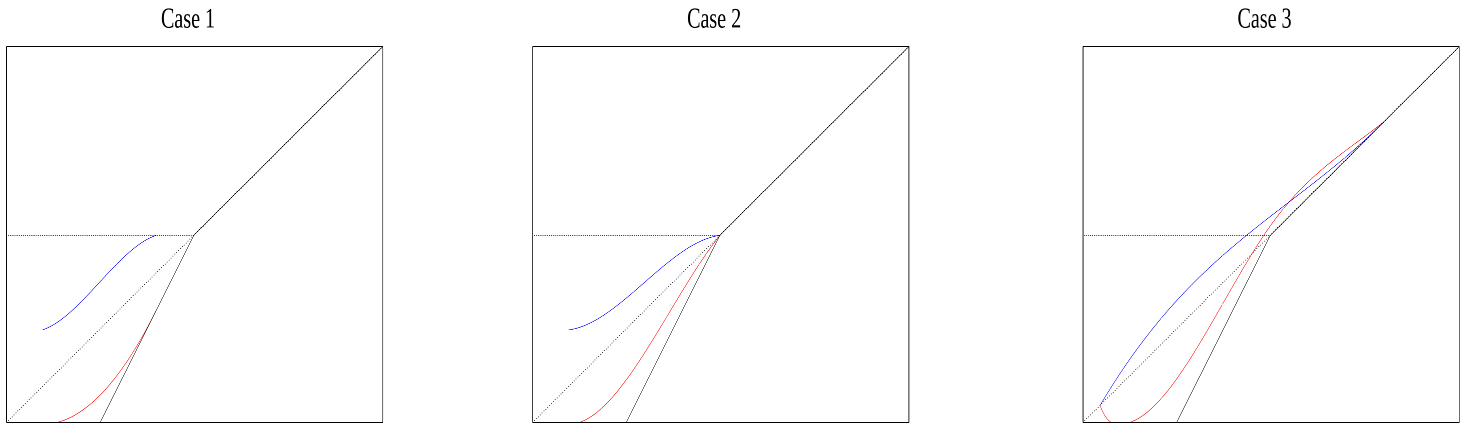

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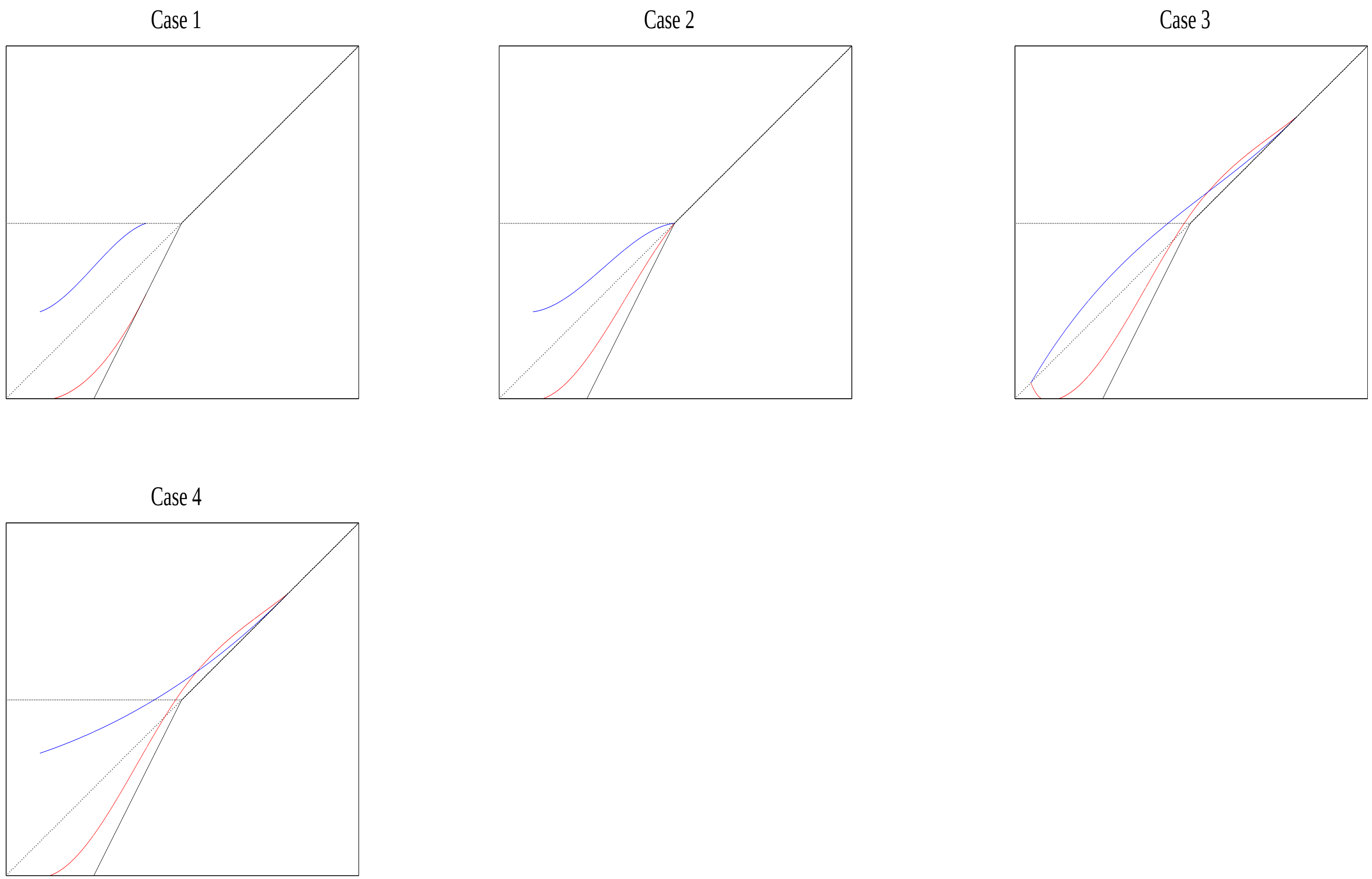

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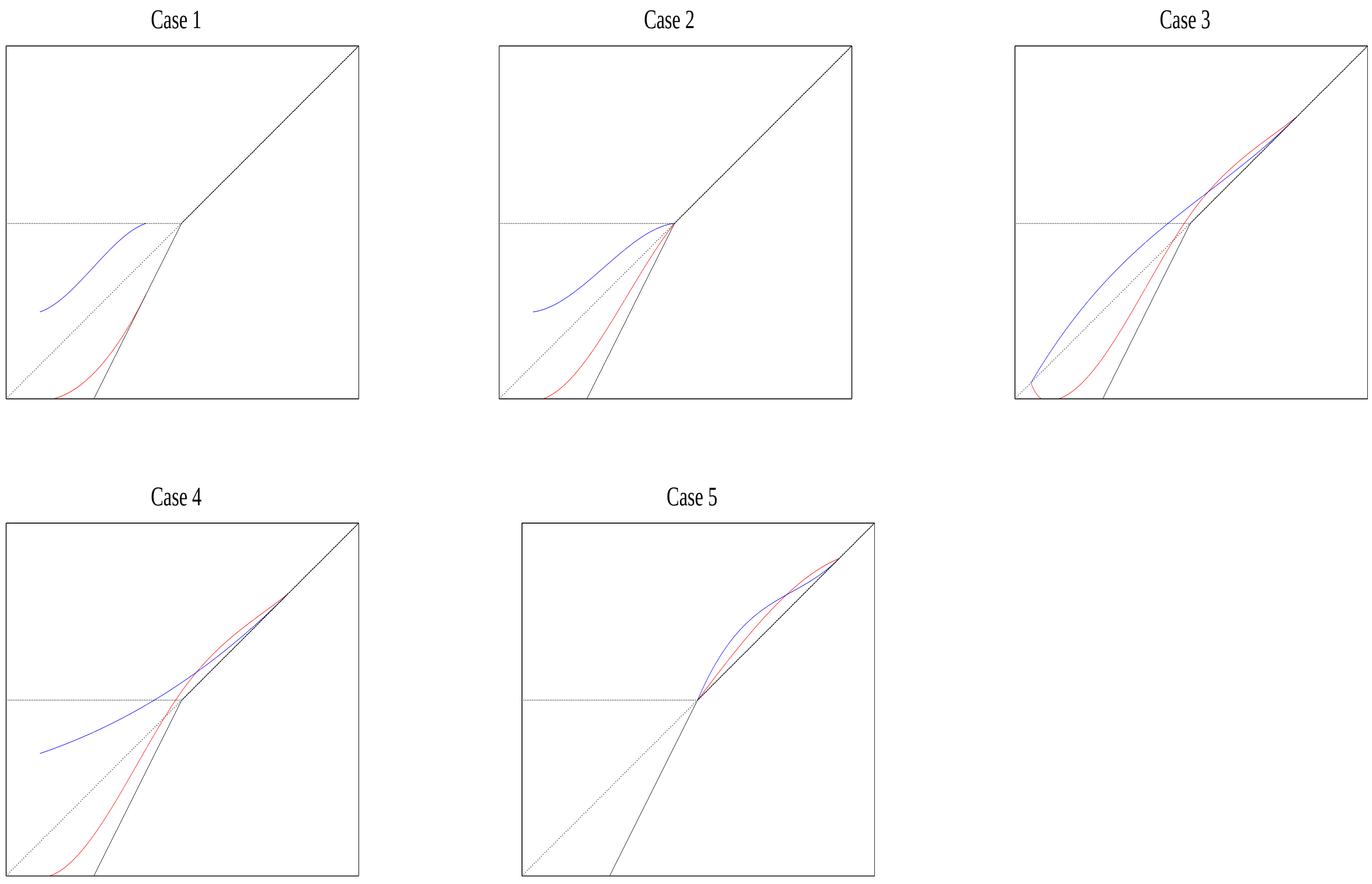

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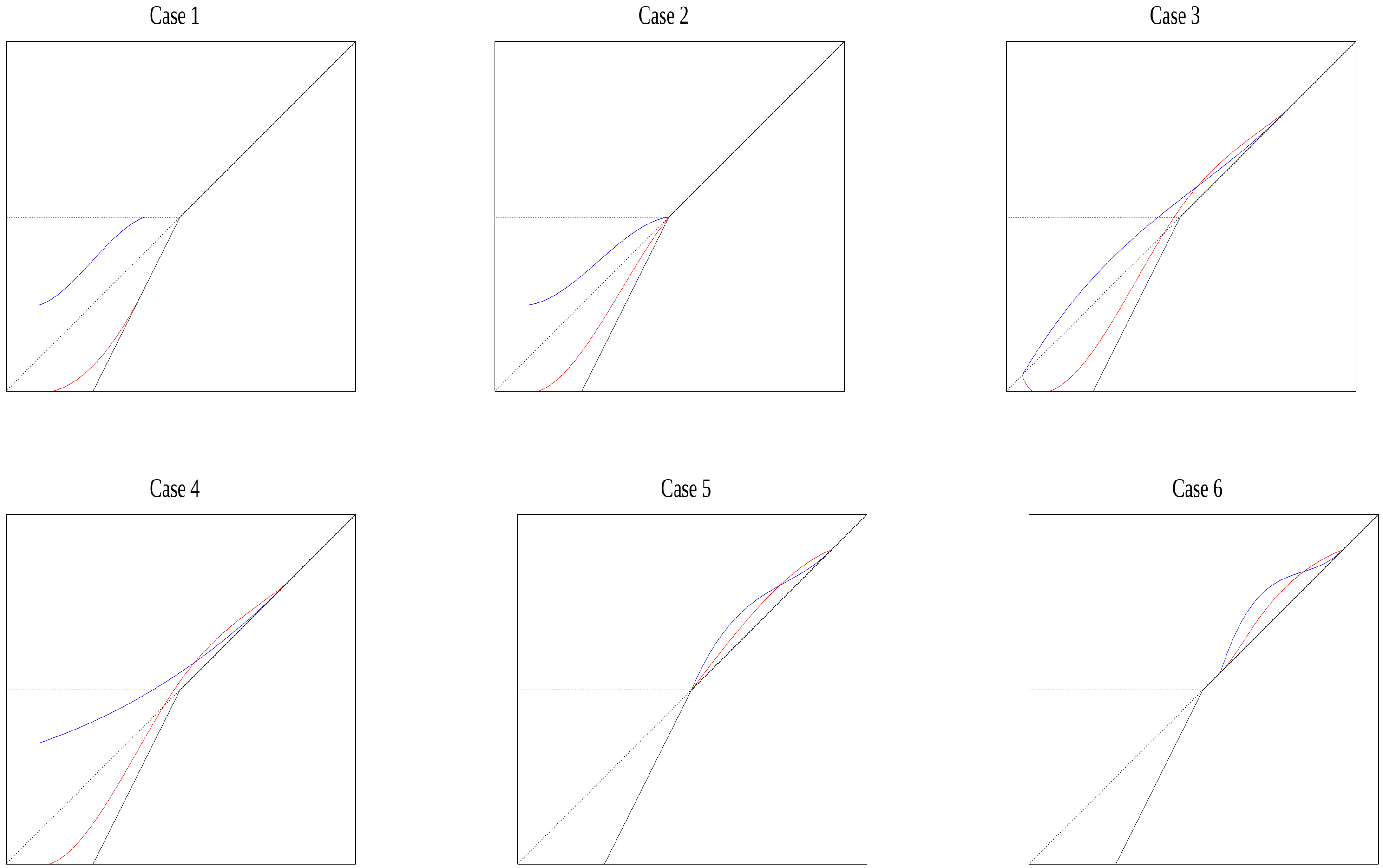

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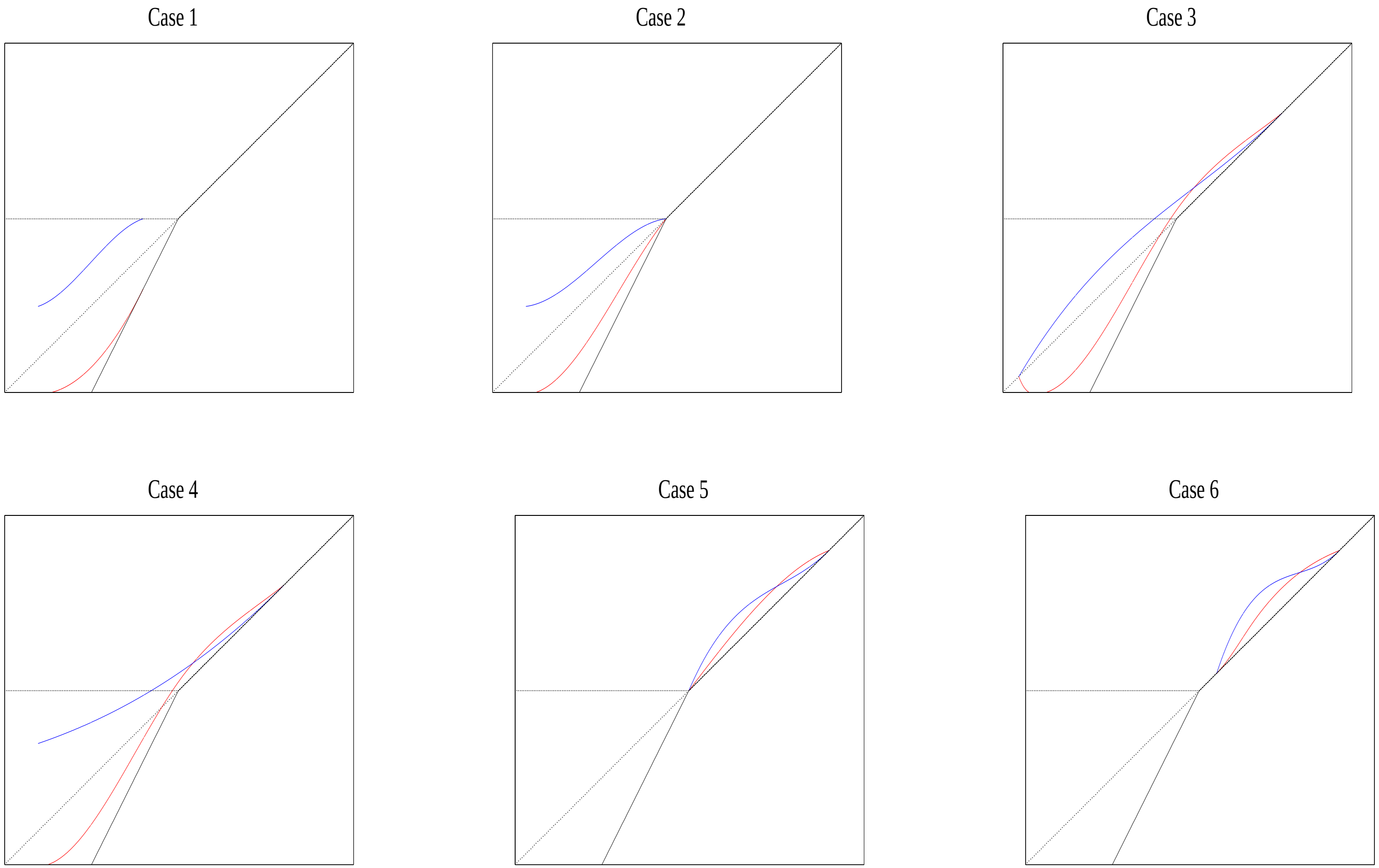

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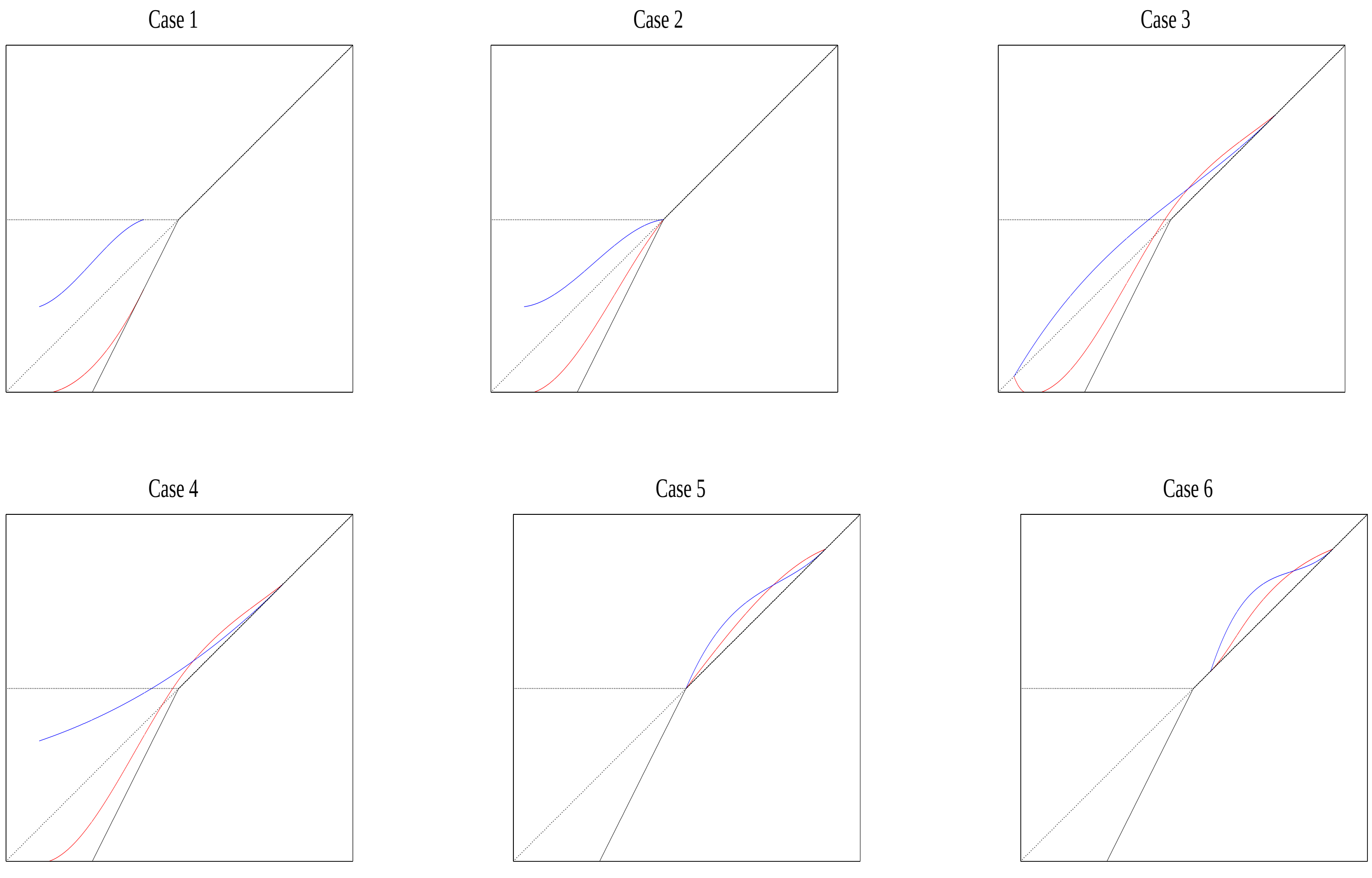

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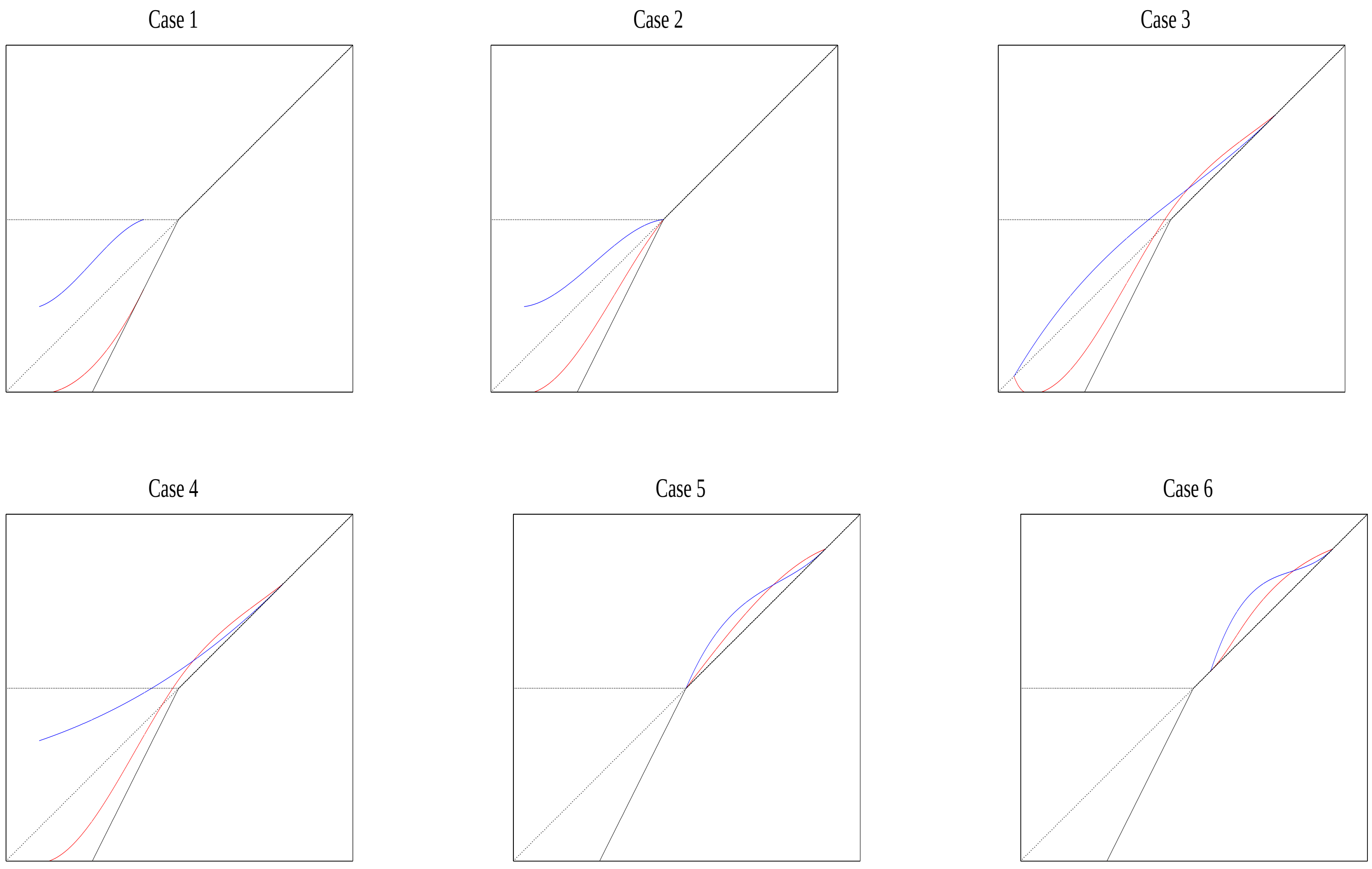

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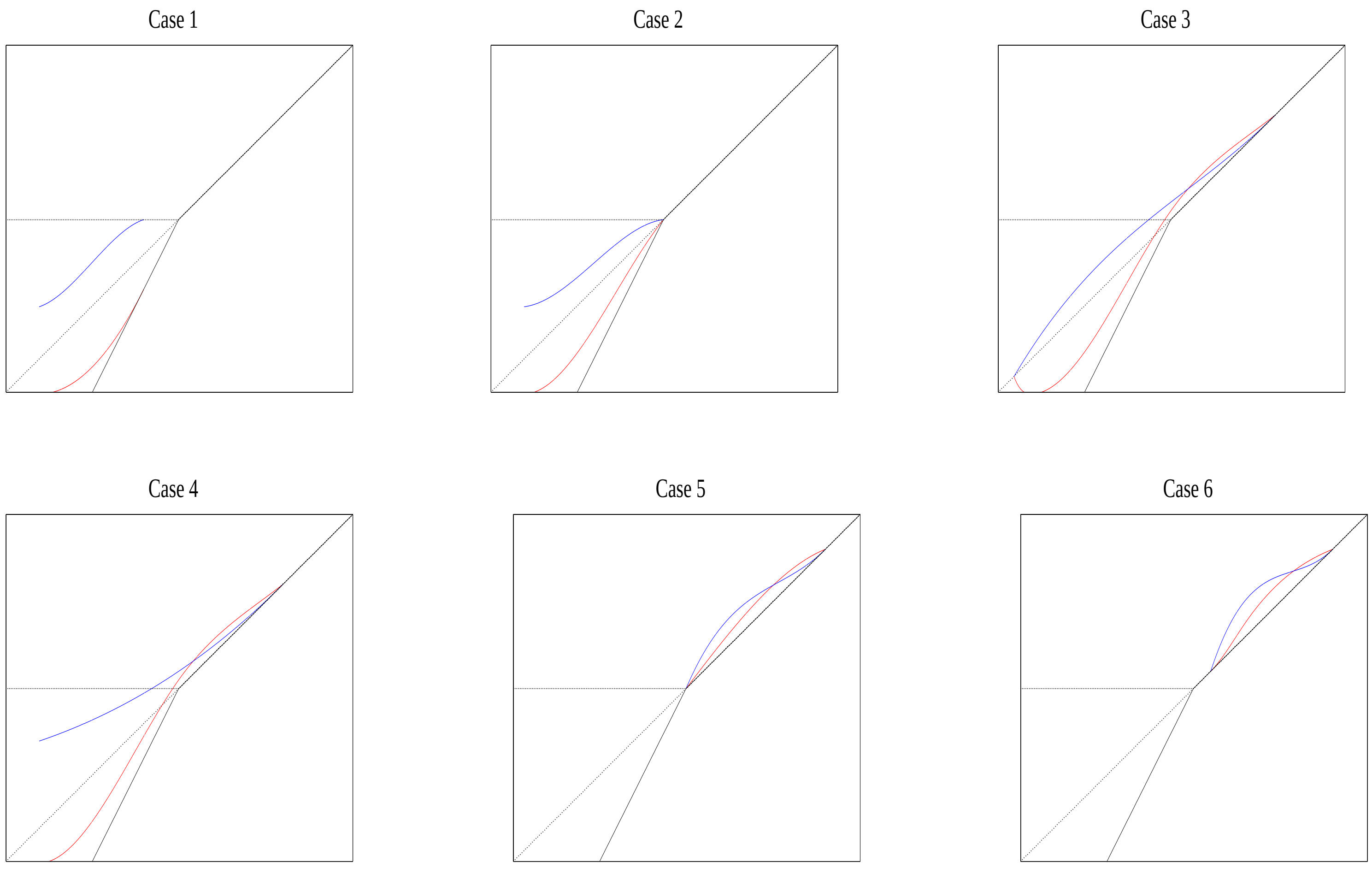

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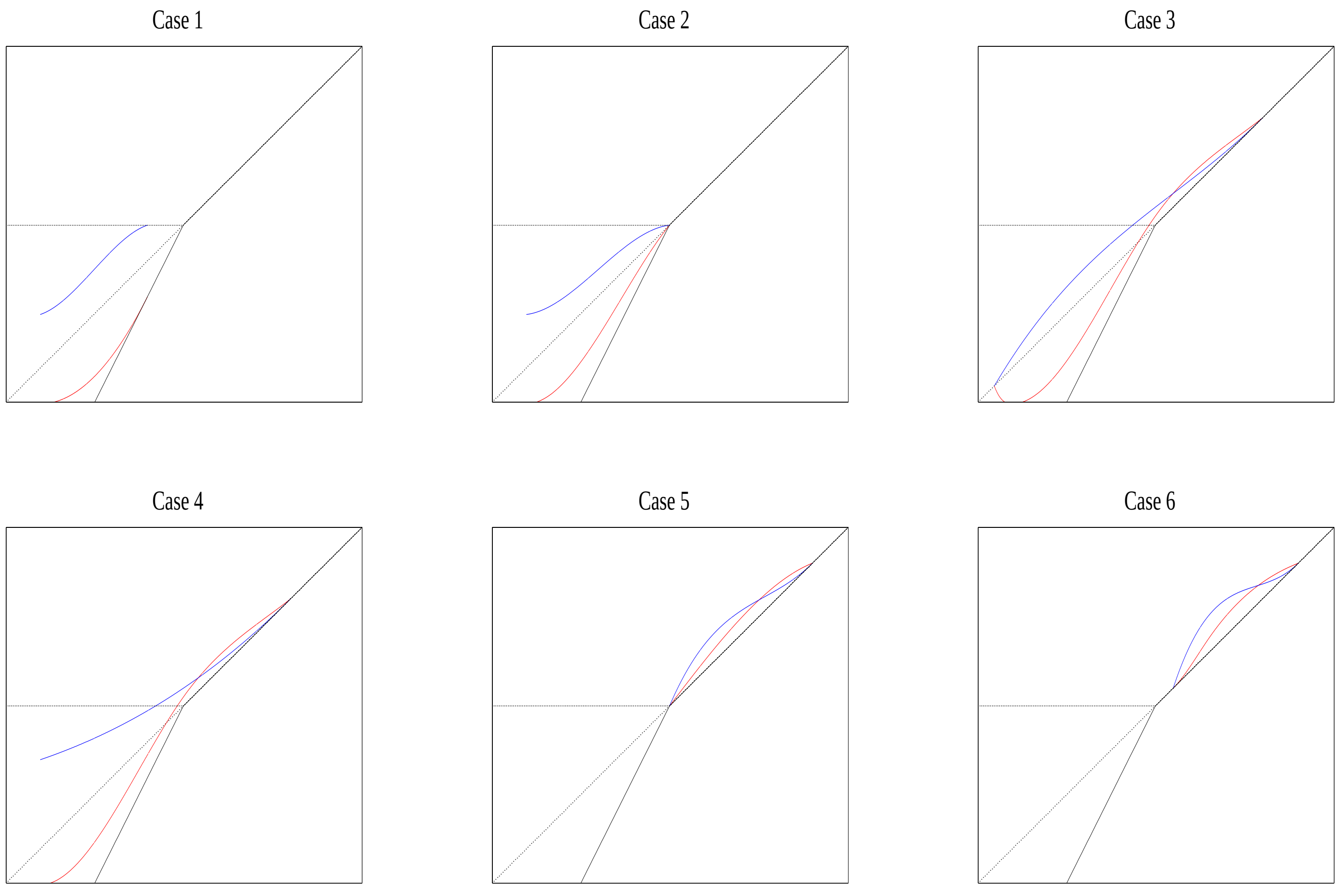
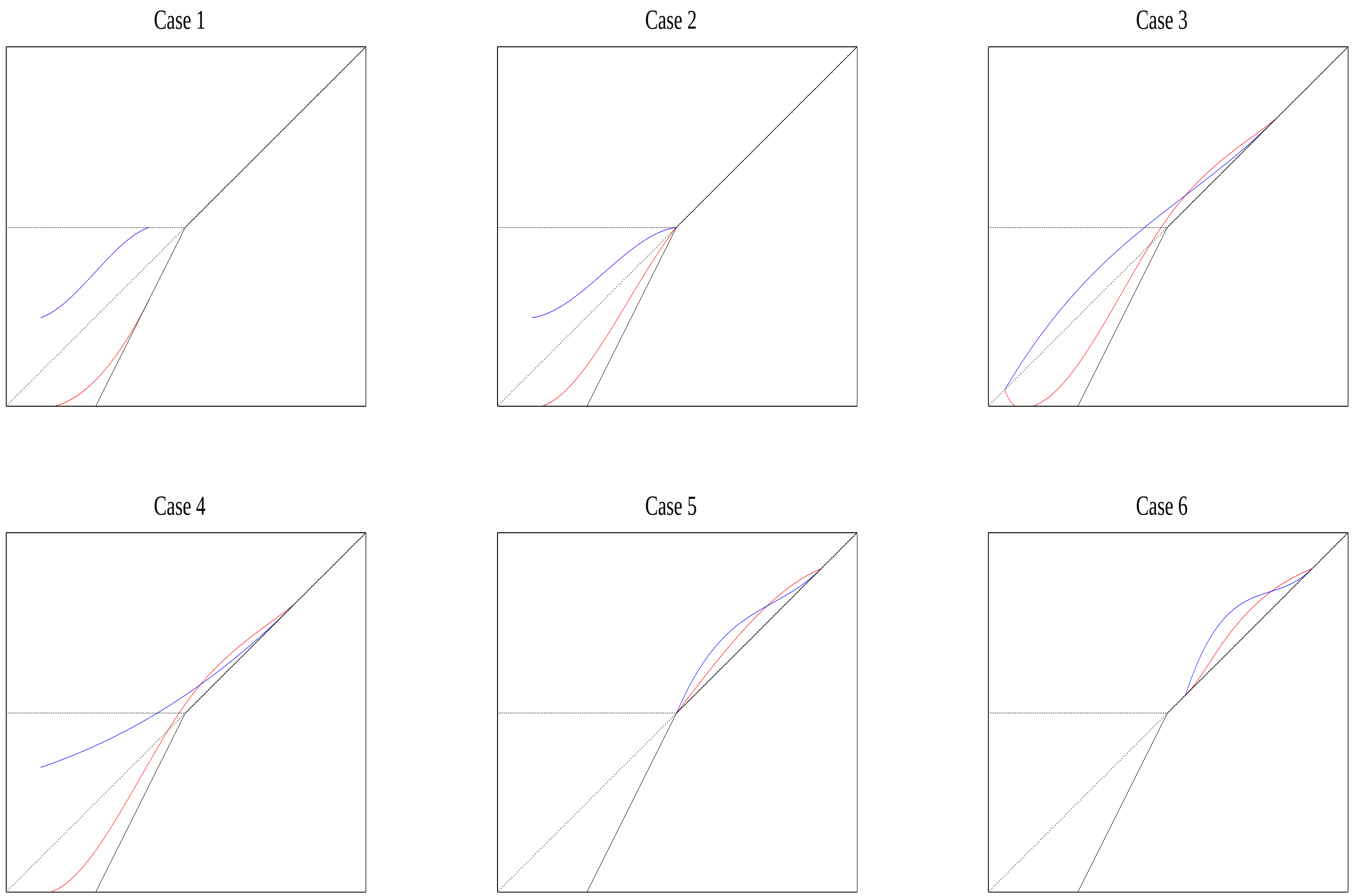
How does it look as $m_0 \downarrow 0$?

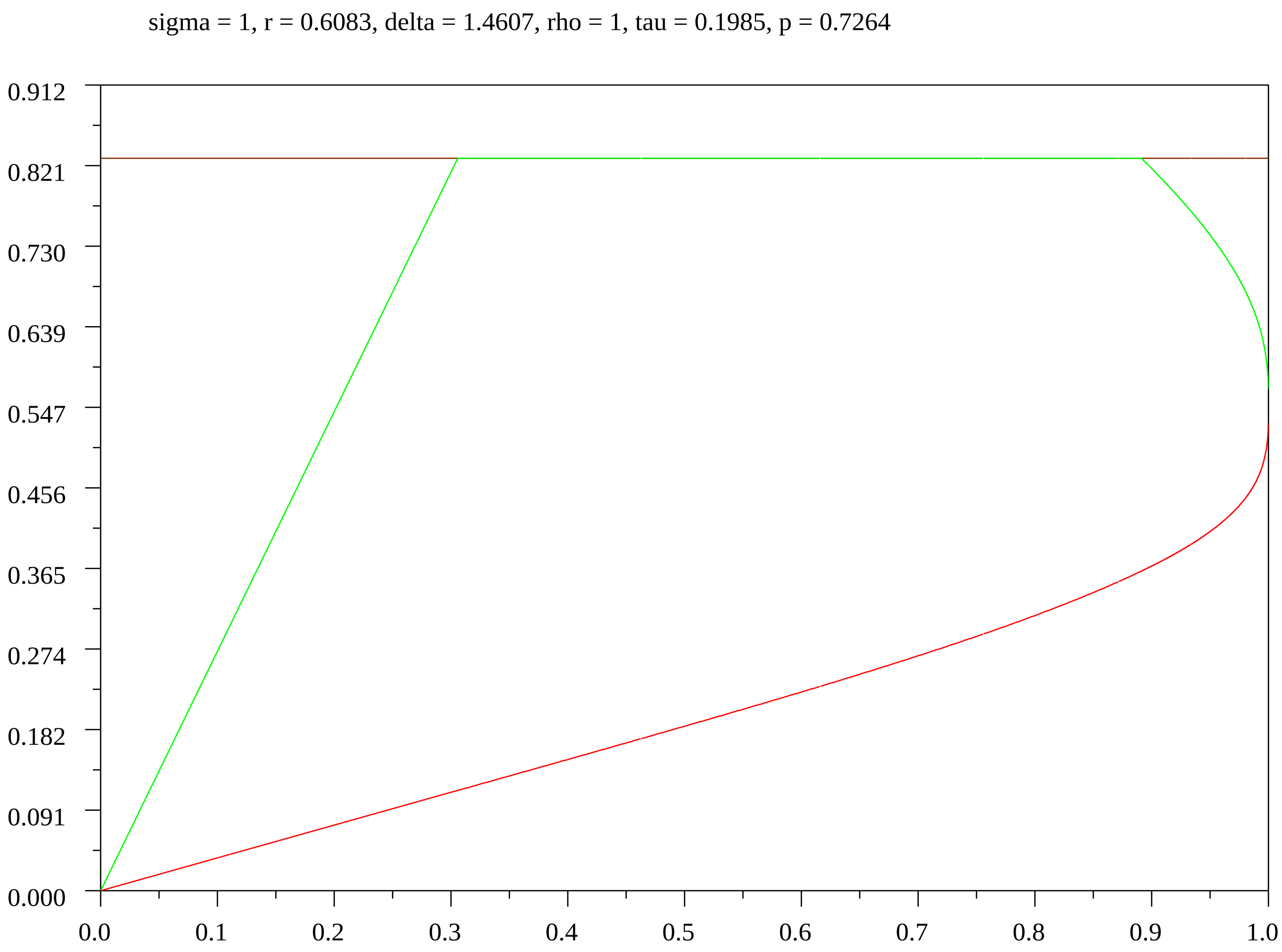


How does it look as $m_0 \downarrow 0$?



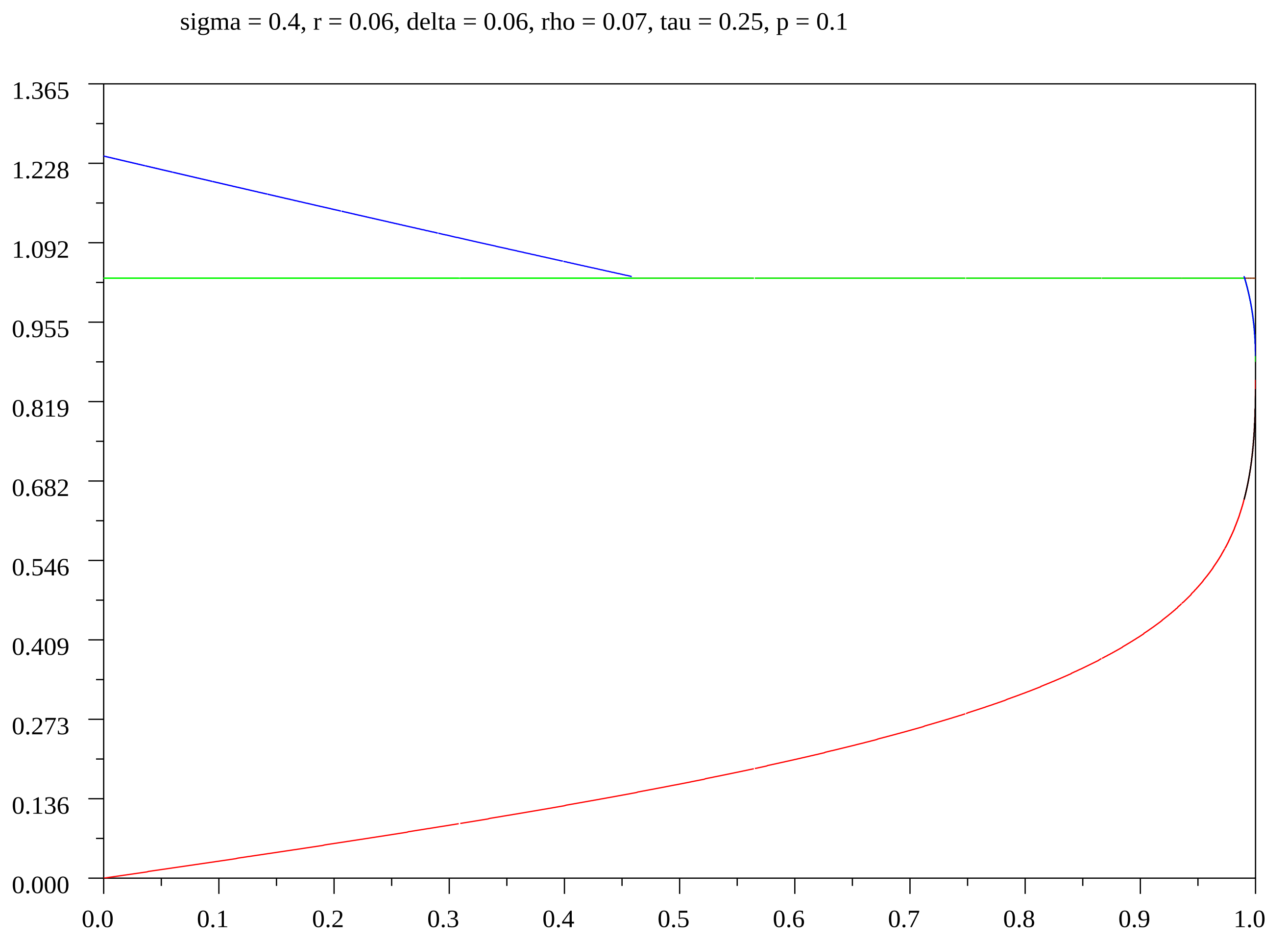
As $m_0 \downarrow 0$, Cases 3 and 4 eventually disappear; so only possible live intervals at $m = 0$ are examples of the others.

Example 13



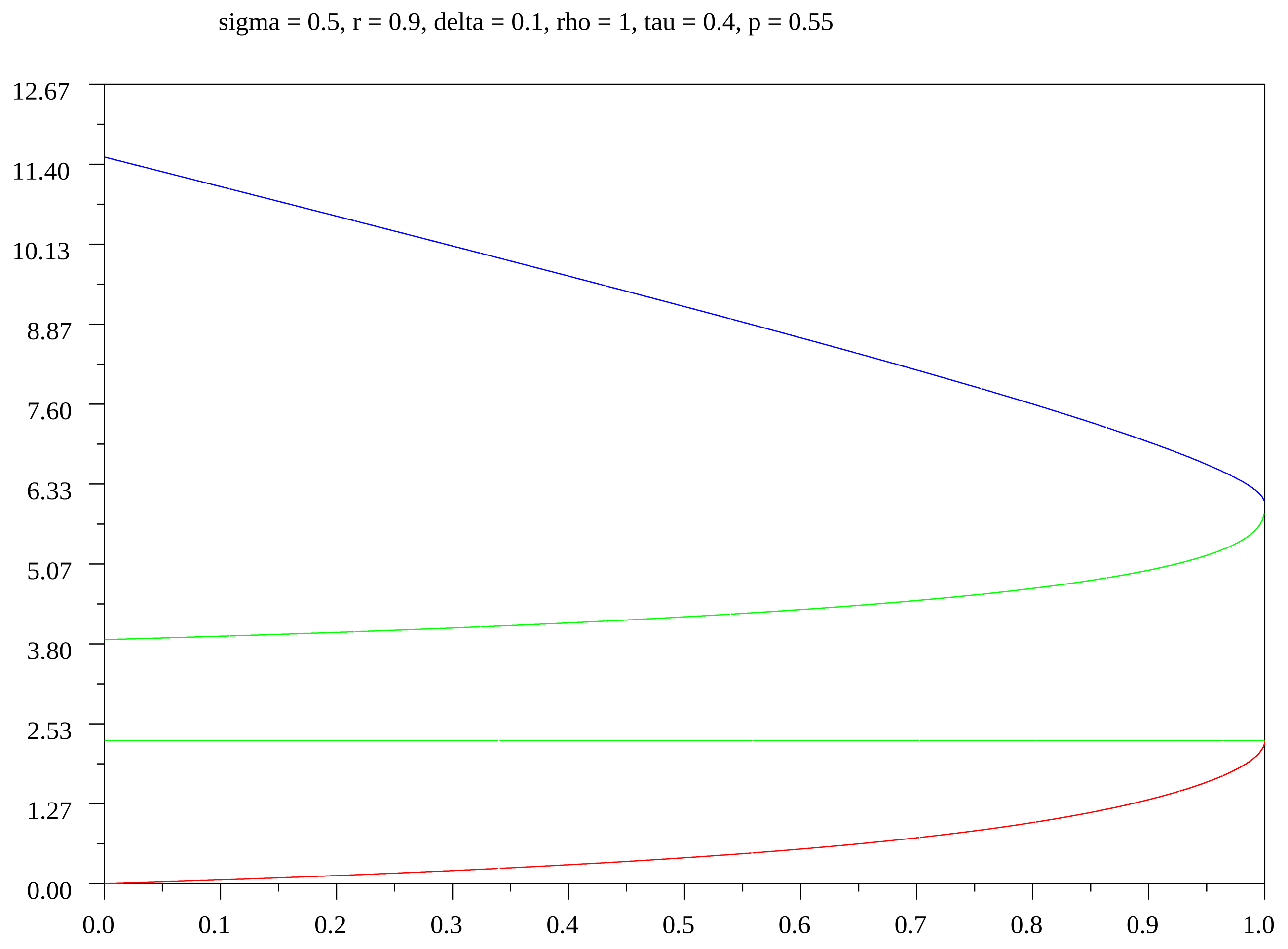
sigma = 1, r = 0.6083, delta = 1.4607, rho = 1, tau = 0.1985, p = 0.7264
0.912
0.821
0.730
0.639
0.547
0.456
0.365
0.274
0.182
0.091
0.000
0.0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1.0

Example 12



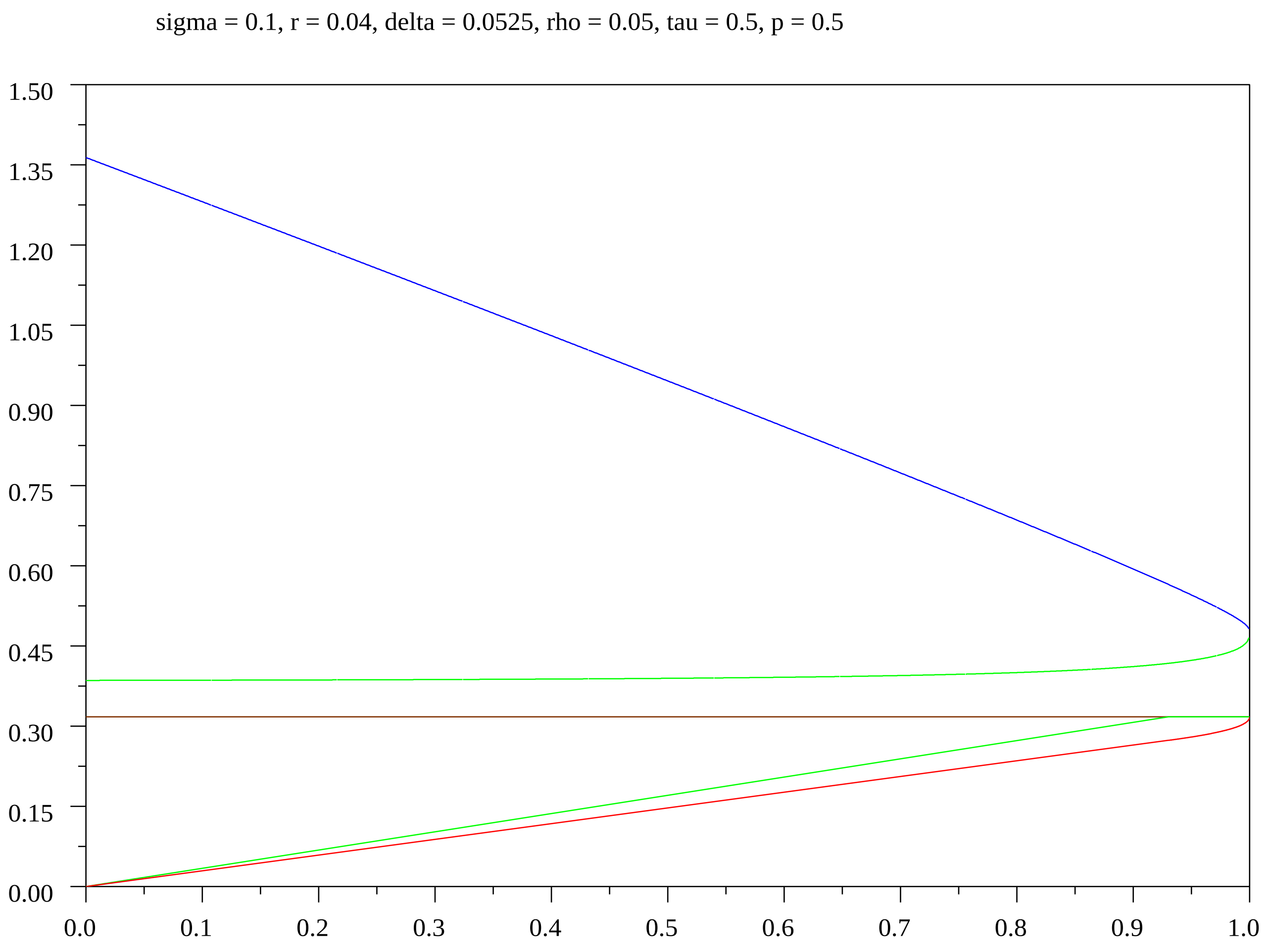
sigma = 0.4, r = 0.06, delta = 0.06, rho = 0.07, tau = 0.25, p = 0.1
1.365
1.228
1.092
0.955
0.819
0.682
0.546
0.409
0.273
0.136
0.000
0.0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1.0

Example 8



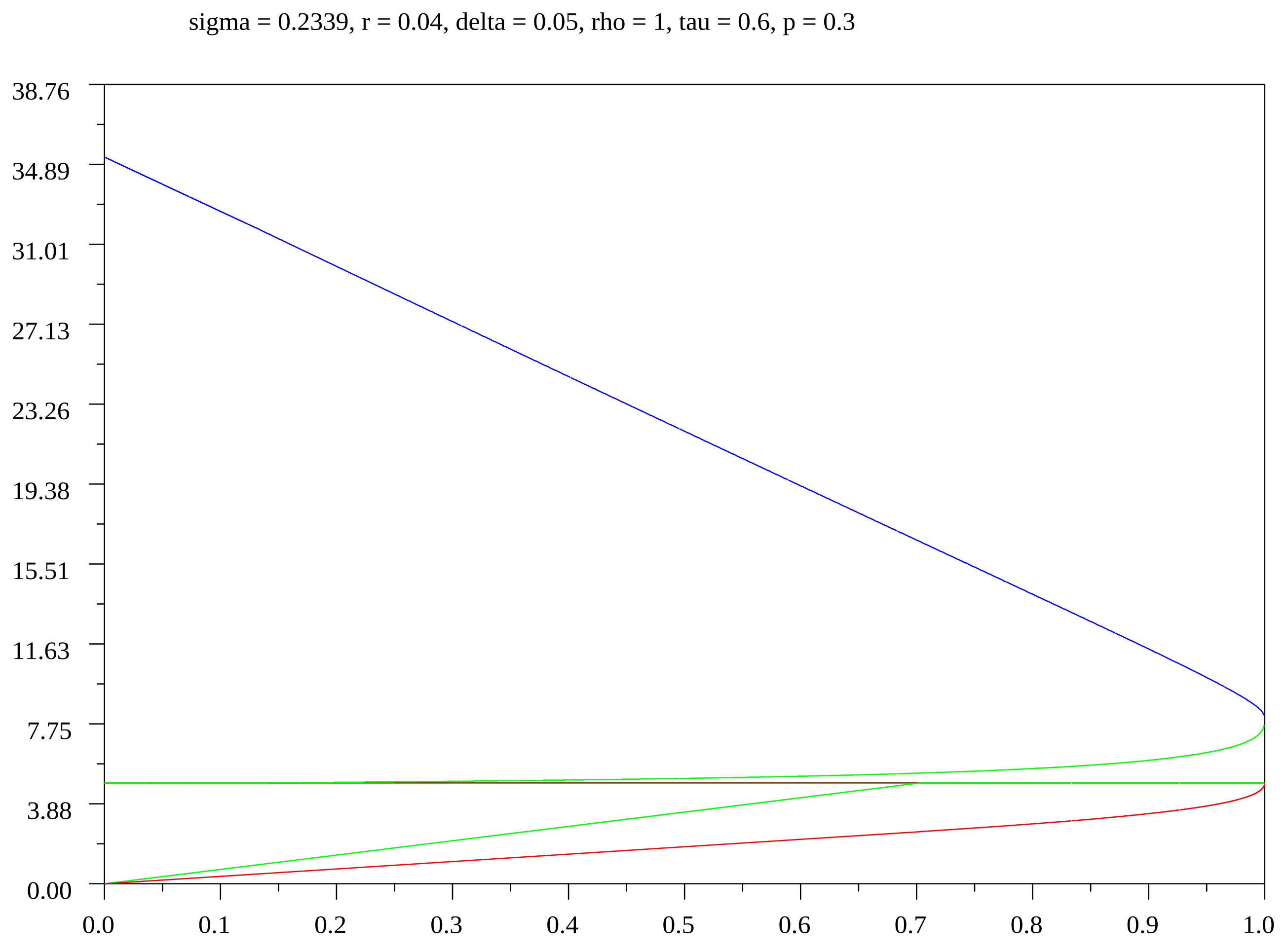
sigma = 0.5, r = 0.9, delta = 0.1, rho = 1, tau = 0.4, p = 0.55
12.67
11.40
10.13
8.87
7.60
6.33
5.07
3.80
2.53
1.27
0.00
0.0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1.0

Example 11



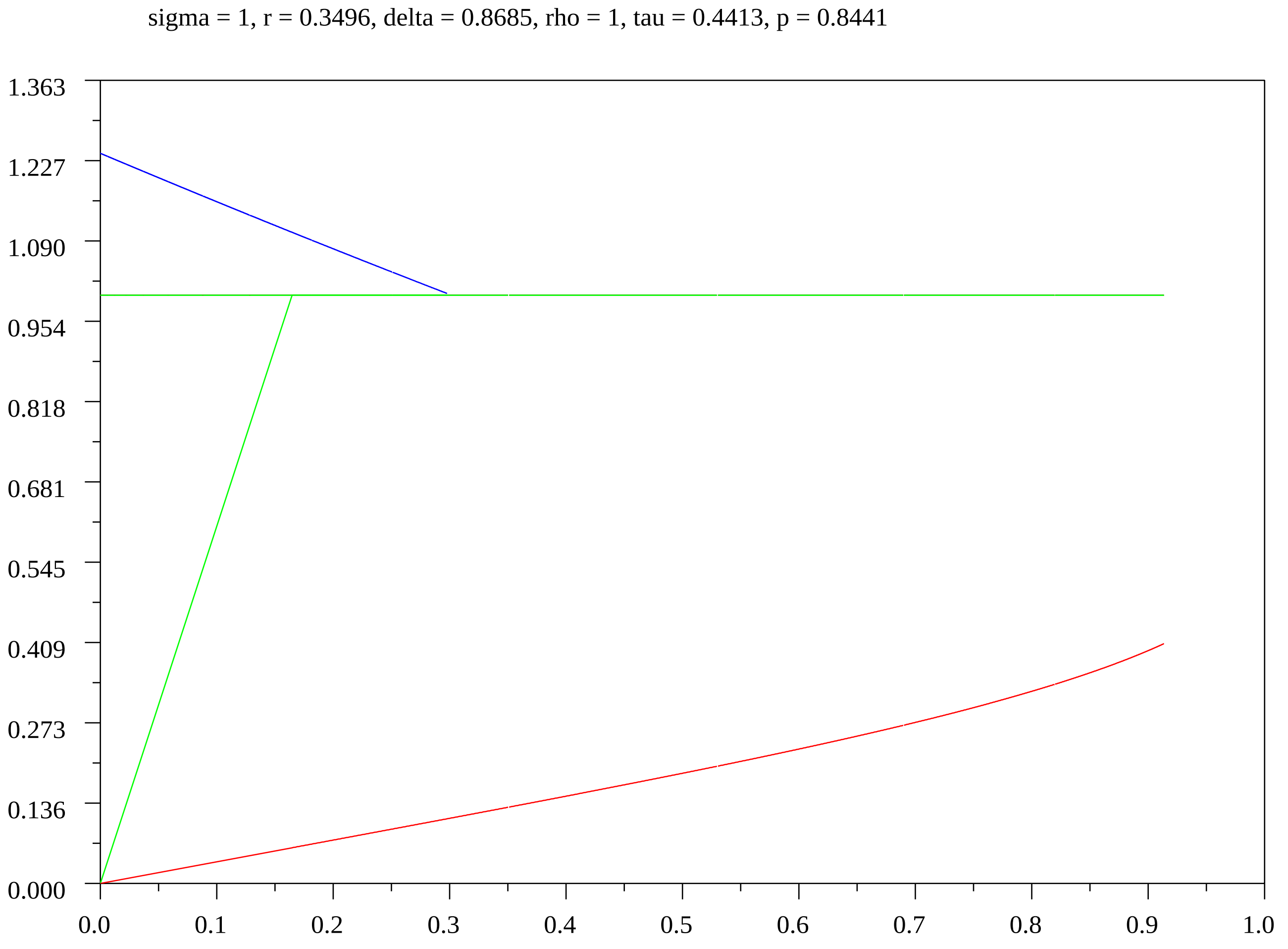
sigma = 0.1, r = 0.04, delta = 0.0525, rho = 0.05, tau = 0.5, p = 0.5
1.50
1.35
1.20
1.05
0.90
0.75
0.60
0.45
0.30
0.15
0.00
0.0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1.0

Example 19



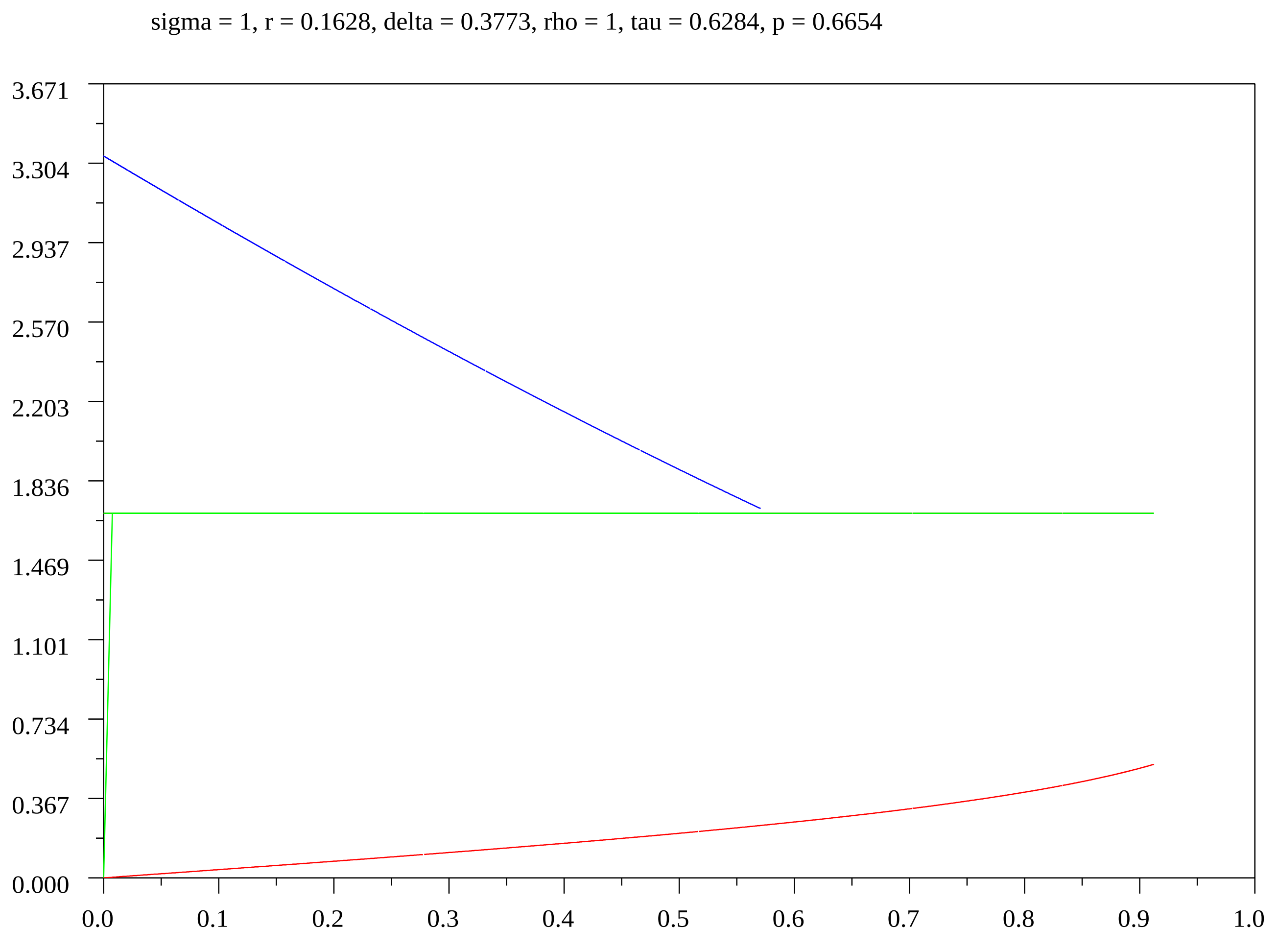
sigma = 0.2339, r = 0.04, delta = 0.05, rho = 1, tau = 0.6, p = 0.3
38.76
34.89
31.01
27.13
23.26
19.38
15.51
11.63
7.75
3.88
0.00
0.0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1.0

Example 30



sigma = 1, r = 0.3496, delta = 0.8685, rho = 1, tau = 0.4413, p = 0.8441
1.363
1.227
1.090
0.954
0.818
0.681
0.545
0.409
0.273
0.136
0.000
0.0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1.0

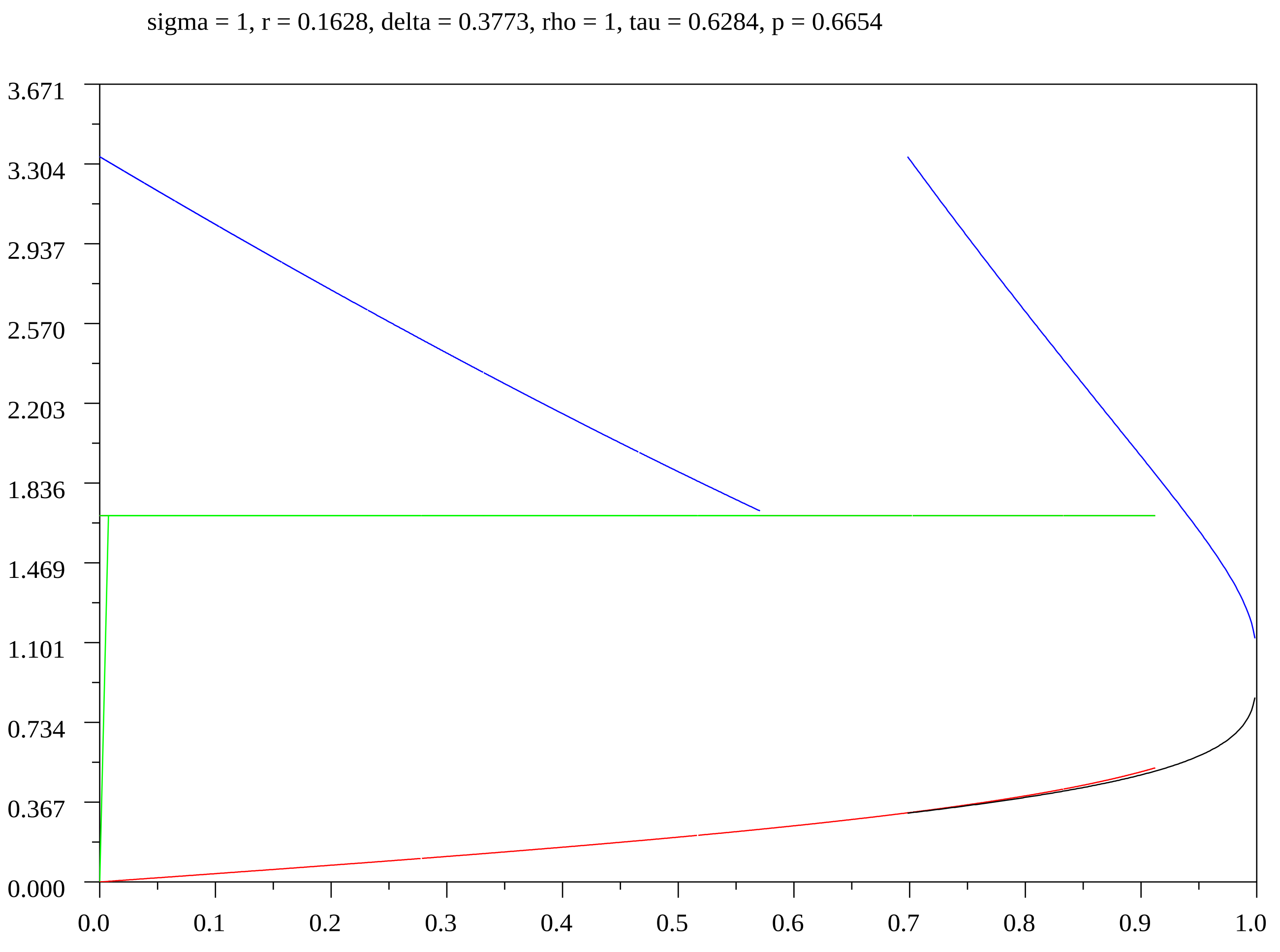
Example 10



sigma = 1, r = 0.1628, delta = 0.3773, rho = 1, tau = 0.6284, p = 0.6654
3.671
3.304
2.937
2.570
2.203
1.836
1.469
1.101
0.734
0.367
0.000
0.0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1.0

Example 10

But Case 4 can arise for larger values of m!!



The case when $K > K_c$

The no-calling solution holds for small m, but as m rises, what do we see?

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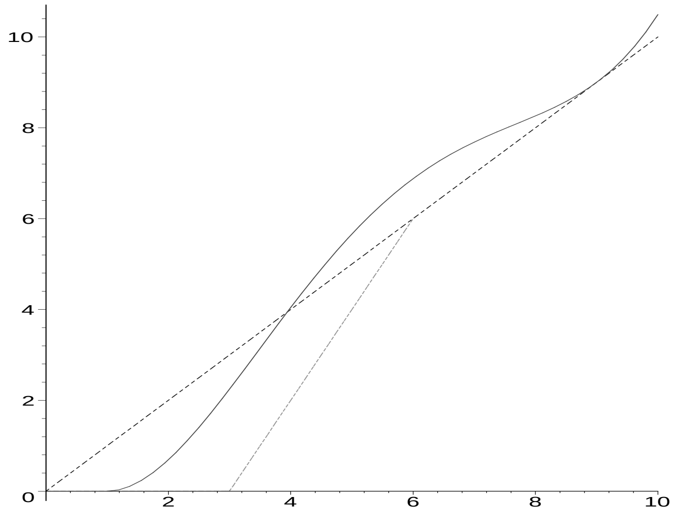
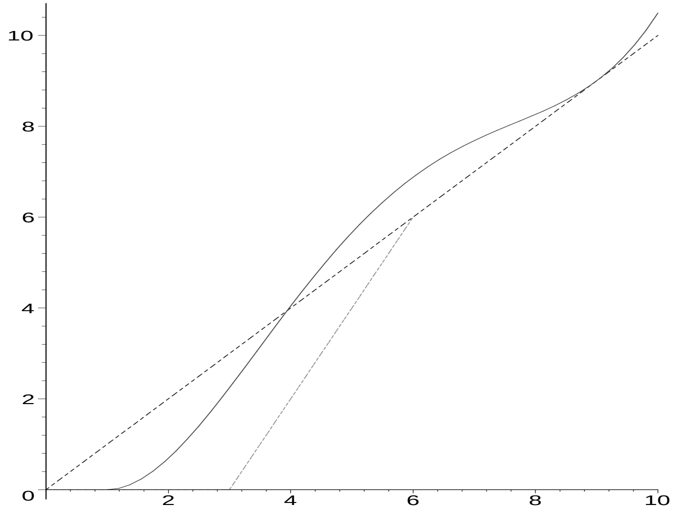
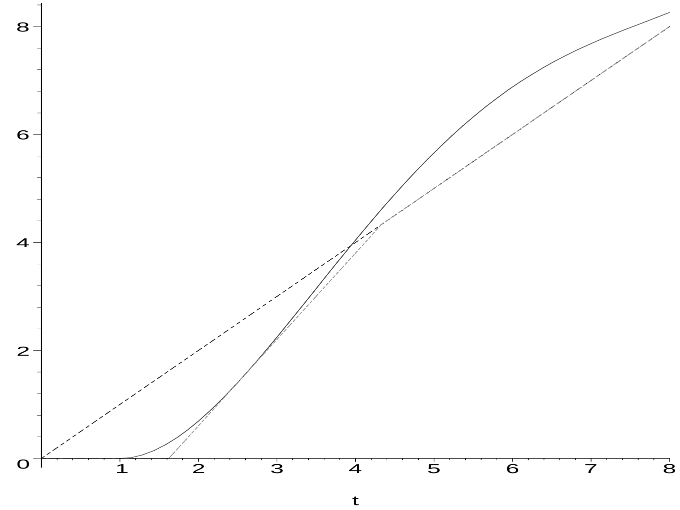


The case when $K > K_c$

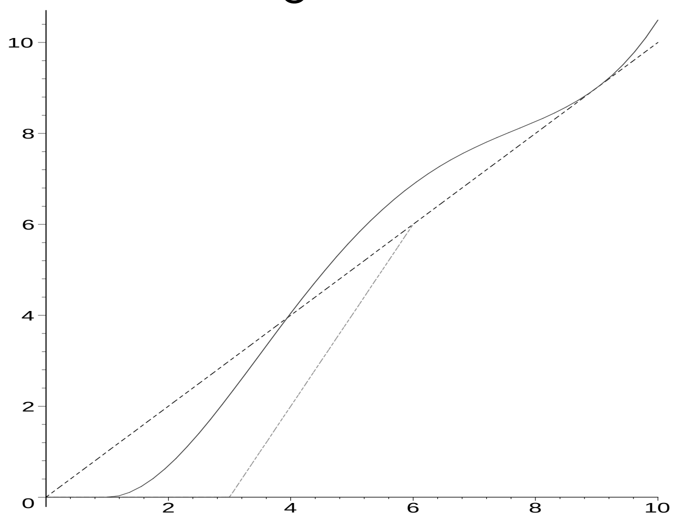
The no-calling solution holds for small m, but as m rises, what do we see?

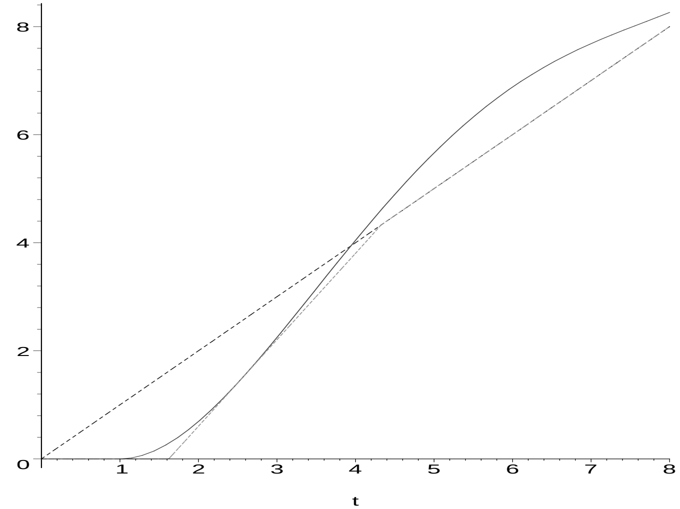


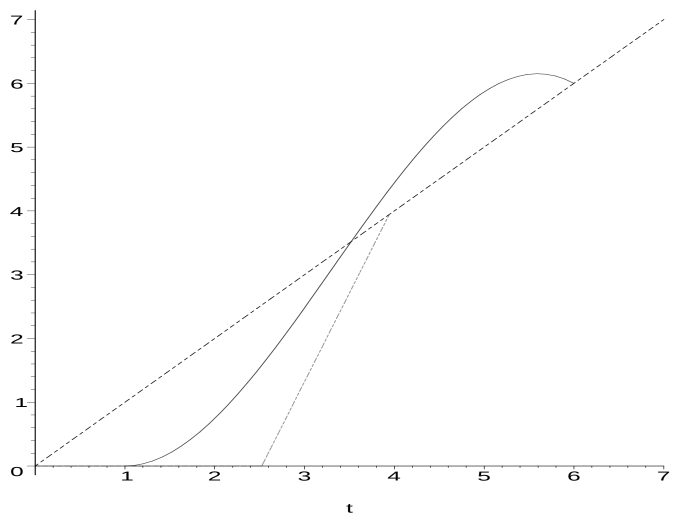


The case when $K > K_c$

The no-calling solution holds for small m, but as m rises, what do we see?

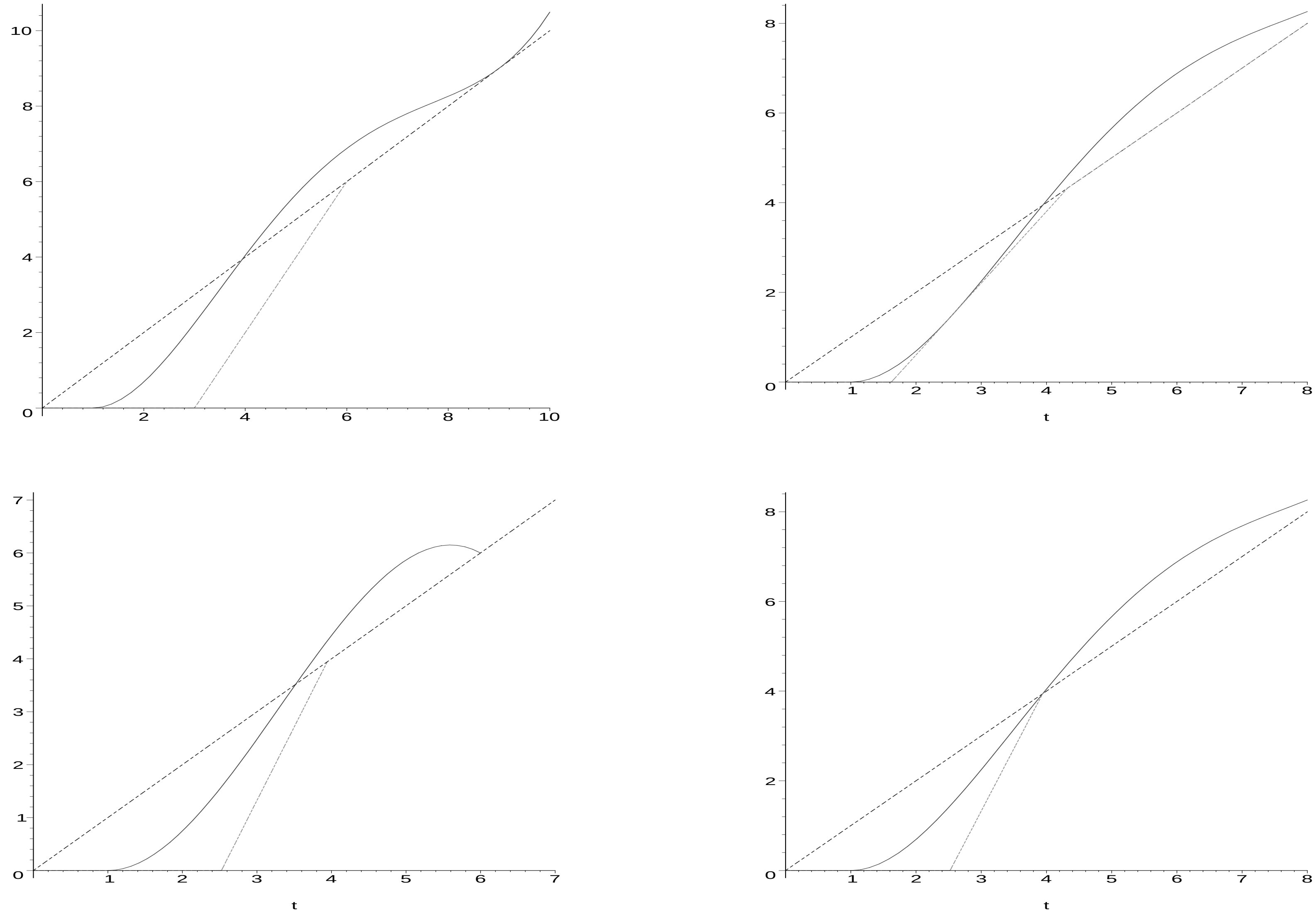






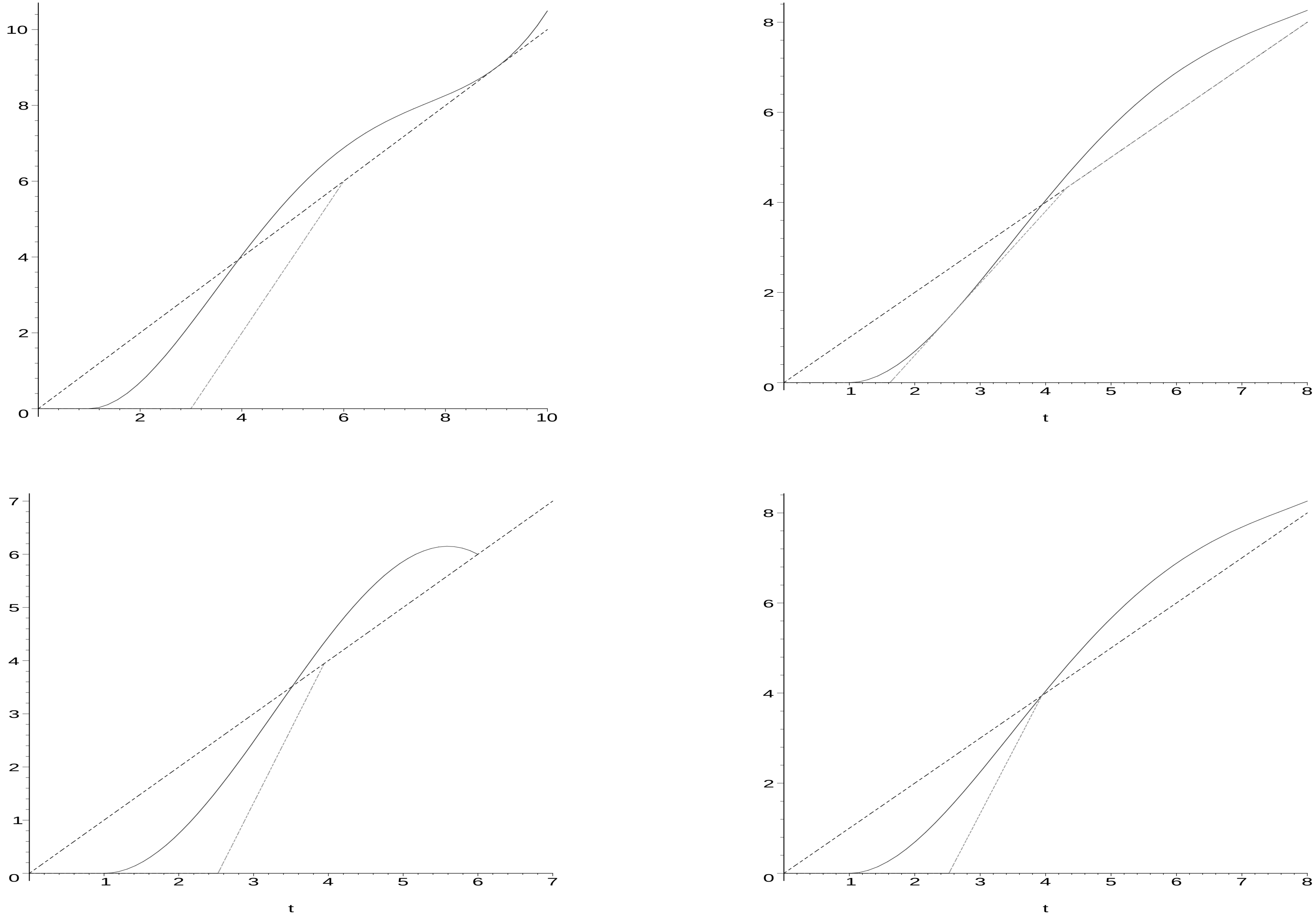
The case when $K > K_c$

The no-calling solution holds for small m, but as m rises, what do we see?



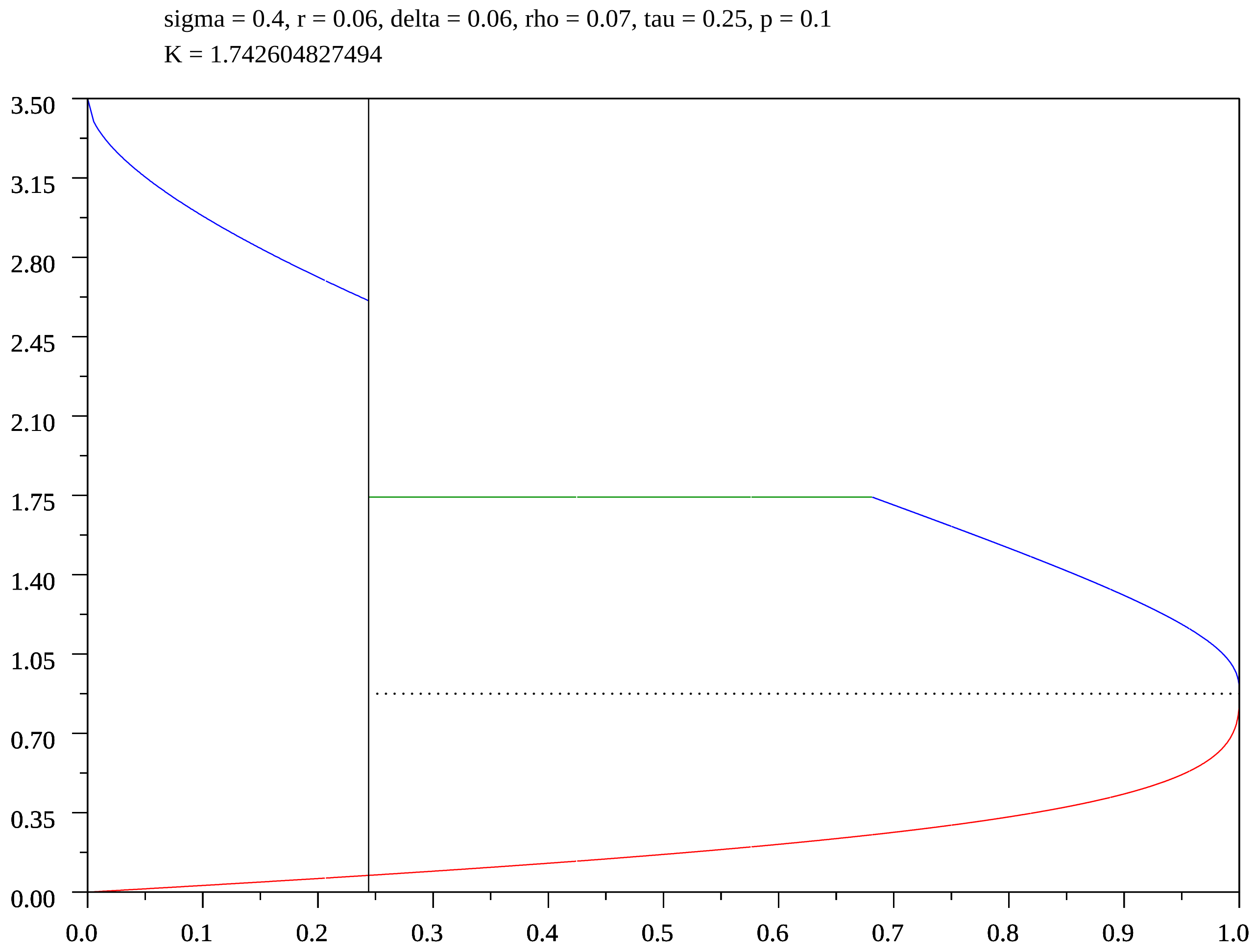
The case when $K > K_c$

The no-calling solution holds for small m, but as m rises, what do we see?

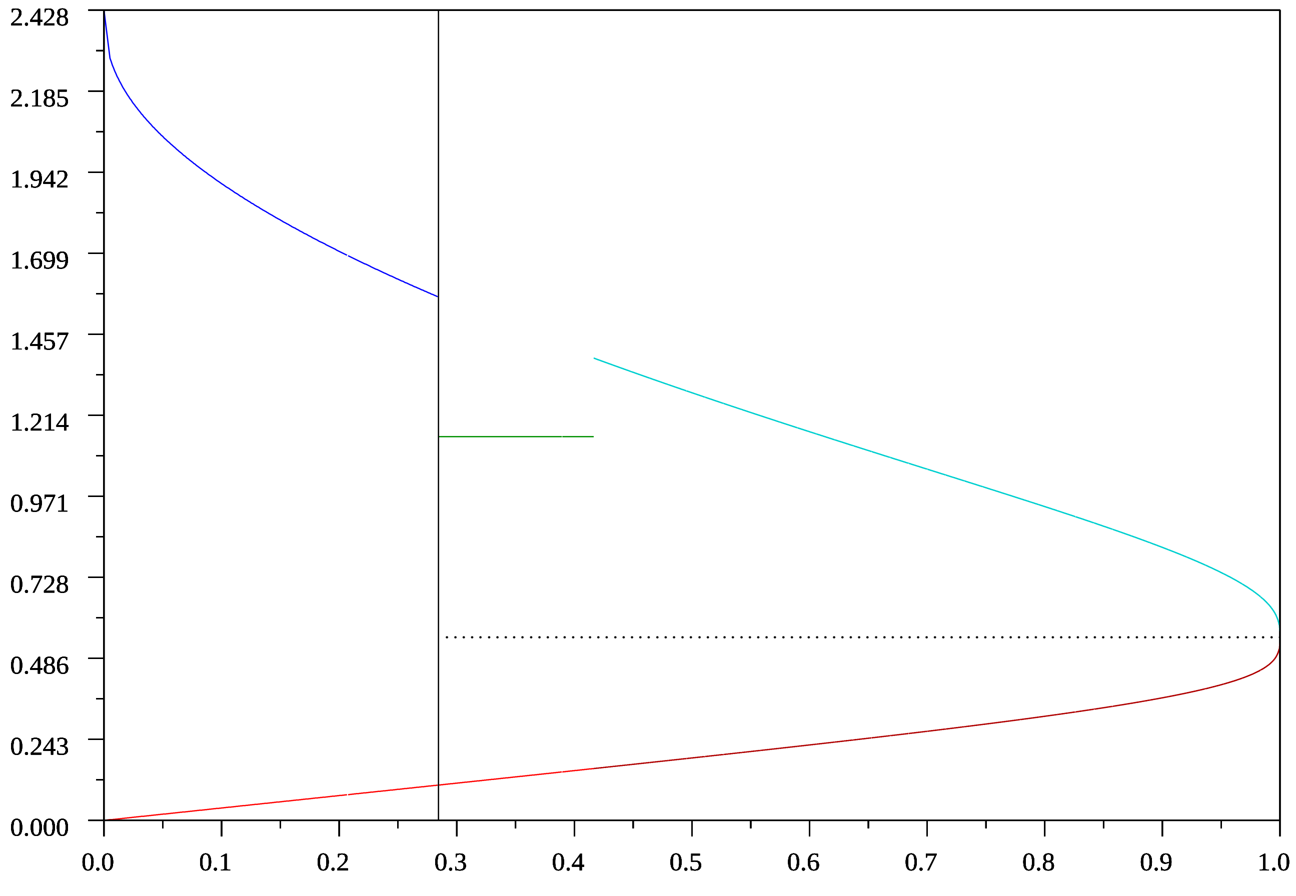


In fact, only the *last* of these ever happens.

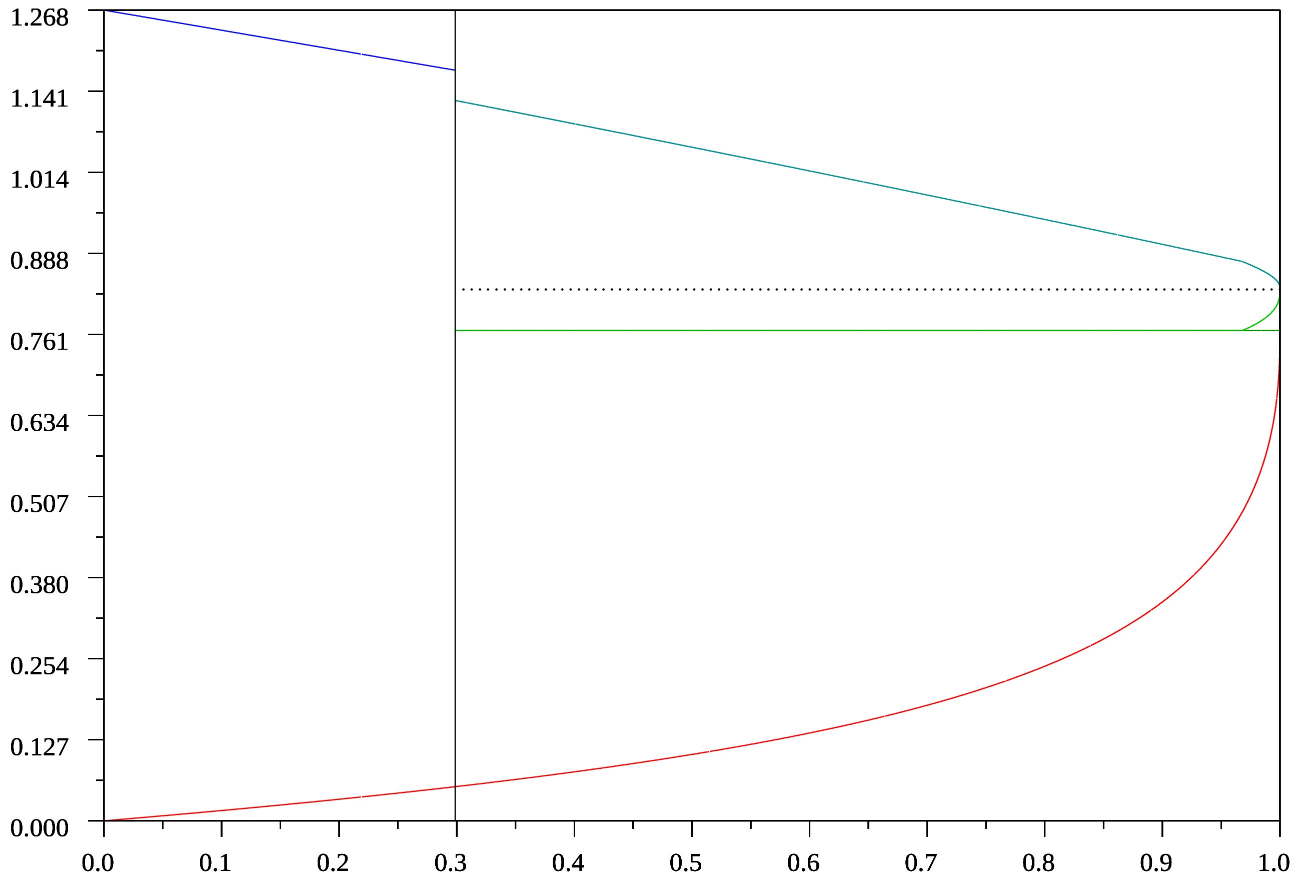
What happens beyond the critical point?



sigma = 1, r = 0.6083, delta = 1.4607, rho = 1, tau = 0.1985, p = 0.7264
K = 1.149641917507



sigma = 1, r = 4.2, delta = 0.9043, rho = 1, tau = 0.2483, p = 0.2283
K = 0.766566211071



CONCLUSIONS

- Ingersoll's paradox: may be explained by tax/cheap loan effect

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  - block conversion
- With calling and high call price, can see
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  - continuous conversion for large enough m
  - calling as S and B fall
  - other types of behaviour