Equity with Markov-modulated dividends

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Abstract

We introduce a simple model for the pricing of European style options when the underlying dividend process is given by a Geometric Brownian Motion with Markov-modulated coefficients. It turns out that the correspondent stock process is characterized by both stochastic coefficients and jumps. Transform methods are used to recover option prices. The models is calibrated to market data and the results compared to some well known stochastic volatility model.

1 Introduction

The Black-Scholes model for a stock is so commonly used as a starting point for analyses of derivative pricing, or optimal investment, that it is easy to miss the

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point that the price of an equity is a *derived* object, not a fundamental. Indeed, arbitrage-pricing theory tells us that the price of a stock is the net present value (NPV) of all future dividends, so once the dividend process has been specified (in the appropriate pricing measure) the price of the stock then follows. Though the Black-Scholes model is not normally presented in terms of a dividend process, it certainly can be (as we shall explain later), and this way of looking at the model can be extremely effective. As an example, we would cite the recent work of Korn & Rogers [KR] on options on stocks paying discrete dividends. By directly modelling the (discrete) dividend process of the stock, this paper is able to come up with a simple way of treating discrete dividends, free from inconsistencies, where various 'industry' modifications of the basic Black-Scholes equation have failed.

As we shall see, the Black-Scholes model with a constant dividend rate arises if we model the dividend process as a geometric Browniam motion in the pricing measure; so if we take *this* model for the dividend process, we will come up with nothing new. What we shall do instead is to take for the dividend process a *Markov-modulated* geometric Brownian motion, and derive the resulting dynamics of the stock price from an equilibirum analysis for a CRRA representative agent. Of course, the Black-Scholes model is a special case. Markov-modulated dynamics have been used before in financial modelling (see, for example Chourdakis [C] and Jobert-Rogers [JR]); they offer simple generalisations of the standard Black-Scholes model, just what is needed for applications.

In Section 2 we present the basic equilibrium model, and use it to derive the state-price density process, and hence the price of the stock. We shall suppose that the dividend process is modulated by a finite-state irreducible Markov chain; an interesting feature is that the price of the share jumps every time the underlying Markov chain jumps, in a way that is endogenously determined, rather than exogenously imposed. We also show how the Black-Scholes model results in the special case where the Markov chain takes only one value.

Next we go on in Section 3 to compute the prices of European put options; in contrast to the benchmark Black-Scholes model, there is no closed-form expression, and we have to resort to numerical methods (transform inversion) to evaluate option prices. This uses the method of Hosono [Ho], and Abate & Whitt [AW] to invert Laplace transforms.

The next stage is to calibrate the model (assuming a 2-state chain) to put option prices on the S&P 500 index. Fitting to three expiries and a range of moneyness, we find a calibrated model that fits data substantially better than various other non-Black-Scholes models in the literature[1], and shows remarkable intertemporal

[1]There is no standard calibration dataset available, so the other models have been calibrated to different datasets from ours.

stability.

2 The equilibrium model

We consider a very simple economy with a single productive asset, a stock, whose output (dividend) process δ obeys

$$\frac{d\delta_t}{\delta_t} = \mu(\xi_t)dt + \sigma(\xi_t)dW_t. \tag{1}$$

Here, μ and σ are deterministic functions of the finite-state Markov chain ξ, with Q-matrix Q, and W is a one dimensional standard Brownian motion independent of the Markov chain ξ.

In this economy, there is a single (representative) agent, who derives utility from consumption, and who owns the stock at time 0. This agent wishes to maximise his objective

$$E\left[\int_0^\infty e^{-\rho t}U(c_t)dt\right],$$

where the consumption process c must be chosen to satisfy the budget constraint

$$\zeta_0^{-1}E\left[\int_0^\infty \zeta_t c_t dt\right] = \zeta_0^{-1}E\left[\int_0^\infty \zeta_t \delta_t dt\right], \tag{2}$$

where ζ is the state-price density process. The left-hand side of (2) is the time-0 value of the agent's consumption process, whereas the right-hand side is the time-0 value of all the future dividends of the stock, that is to say, the price S_0 at time 0 of the stock. More generally, we have the price at time t of the stock is expressed as

$$S_t = \zeta_t^{-1}E_t\left[\int_t^\infty \zeta_u \delta_u du\right]. \tag{3}$$

It is well known (see, for example, Breeden [B], or Karatzas & Shreve [KS]) that the optimal consumption c^* is related to the state-price density process by

$$e^{-\rho t}U'(c_t^*) = \lambda\zeta_t \tag{4}$$

for some positive constant λ. Thus if markets are to clear, that is, $c^* = \delta$, we may use (4) to deduce what the state-price density process must be, up to an irrelevant positive constant.

For simplicity and tractability, we shall suppose that the agent has a CRRA utility

$$U(x) = \frac{x^{1-R}}{1-R},$$

for some positive R different from 1. In this case, we can express ζ explicitly in terms of δ, using (4) and the market-clearing condition:

$$\zeta_t = \lambda^{-1} e^{-\rho t} \delta_t^{-R}. \tag{5}$$

We may (and shall) without loss of generality suppose that $\lambda = 1$ in all the follows.

Within this very concrete model, it is possible to make the expression (3) for the stock price much more explicit. Indeed, taking $t = 0$ (with no real loss of generality) and using the notation $\tilde{\mu}(x) \equiv \mu(x) - \frac{1}{2}\sigma(x)^2$, we have

$$\begin{aligned}
S_0 &= \zeta_0^{-1} E_0 \left[\int_0^\infty \zeta_t \delta_t dt \right] \\
&= \delta_0^R E_0 \left[\int_0^\infty e^{-\rho t} \delta_t^{1-R} dt \right] \\
&= \delta_0 E_0 \left[\int_0^\infty \exp\Big\{ -\rho t + (1-R) \int_0^t \sigma(\xi_s) dW_s + (1-R) \int_0^t \tilde{\mu}(\xi_s) ds \Big\} \, dt \right] \\
&= \delta_0 E_0 \left[\int_0^\infty \exp\Big\{ -\rho t + \int_0^t f(\xi_s) ds \Big\} \, dt \right] \\
&\equiv \delta_0 v(\xi_0),
\end{aligned}$$

say, where

$$f(x) \equiv (1-R)\tilde{\mu}(x) + \frac{1}{2}(1-R)^2 \sigma^2(x). \tag{6}$$

Thus we shall have in general that

$$S_t = \delta_t v(\xi_t), \tag{7}$$

where

$$v(x) = E\left[\int_0^\infty \exp\Big\{ -\rho t + \int_0^t f(\xi_s) ds \Big\} \, dt \,\middle|\, \xi_0 = x \right].$$

Routine methods allow us to express v in terms of the generator Q of the Markov chain:

$$v = (\rho - Q - F)^{-1} \mathbf{1}, \tag{8}$$

where $\mathbf{1}$ is the constant vector all of whose entries are 1, and F is the diagonal matrix of f. We assume that ρ is large enough that all eigenvalues of $\rho - Q - F$ are in the open right half plane.

The explicit form (7) for the stock price shows how jumps of S arise at jump times of the chain ξ. The equilibrium analysis that has led to the equilibrium price (7) for S also allows us to compute the equilibrium riskless rate, and the martingale measure. Indeed, we have

$$\begin{aligned}
\zeta_t &= e^{-\rho t}\delta_t^{-R} \\
&= \exp\{-\rho t - R\int_0^t \sigma(\xi_s)dW_s - R\int_0^t \tilde{\mu}(\xi_s)ds\} \\
&= \exp\{-R\int_0^t \sigma(\xi_s)dW_s - \frac{1}{2}\int_0^t R^2\sigma(\xi_s)^2 ds - \int_0^t r(\xi_s)ds\},
\end{aligned}$$

where

$$r(x) = \rho + R\tilde{\mu}(x) - \frac{1}{2}R^2\sigma^2(x). \tag{9}$$

Under the pricing measure we therefore have

$$\frac{d\delta_t}{\delta_t} = \sigma(\xi_t)dW_t^* + (\mu(\xi_t) - R\sigma(\xi_t)^2)dt, \tag{10}$$

where W^* is a Brownian motion in the pricing measure.

It is also clear from (7) that if $|I| = 1$ then the stock price process will be a geometric Brownian motion with no jumps, and with a dividend process proportional to S:

$$dS = S(\bar{\sigma}dW^* + (r-b)dt) \tag{11}$$

- that is, the Black-Scholes model. To match up the parameters in this familiar specification with those of the equilibrium model, we see from (7) and(10) that

$$\begin{aligned}
\bar{\sigma} &= \sigma, \\
b &= \rho - f \qquad\qquad \text{from (8), (7);} \\
&\equiv \rho - (1-R)(\mu - R\sigma^2/2), \\
r &= \rho + \mu R - R(1+R)\sigma^2/2 \qquad \text{from (9),}
\end{aligned}$$

and the consistency condition $r - b = \mu - R\sigma^2$ from comparing drifts in (10) and (11) is easily checked. There are three parameters in the equilibrium model, ρ, R and μ to fit the two parameters r, b of the Black-Scholes model, so there is one degree of indeterminacy; however, if we require to match also the objective rate of return of the stock μ, then there is a unique choice of R and ρ that will work:

$$R = \frac{r-b-\mu}{\sigma^2}, \tag{12}$$

$$\rho = b + (1-R)(\mu - R\sigma^2/2). \tag{13}$$

There is no guarantee that these values of ρ and R will satisfy the required bounds $\rho > 0$ and $R > 0$, however. There is no contradiction here; an equilibrium model is arbitrage-free, but not every arbitrage-free pricing system need arise from an equilibrium, as this example shows.

3 Option pricing by integral transforms

In this section we show how to use integral transforms to price put options under the model described in the previous sections. It is well known (see, for example, [CM], or [RZ]) that pricing a put option when we only know the cumulant-generating function of the log-stock price[2] is best done by transforming the put price in the log strike, and then numerically inverting the transform.

In more detail, the price of a put option with maturity T is given by

$$P_T(k) := \frac{1}{\zeta_0} E\left[\zeta_T \left(e^k - e^s\right)^+\right], \tag{14}$$

where ζ_t is given by (5), and s and k are the log stock and the log strike respectively. For $\mathrm{Re}(\alpha) > 1$ we define

$$\widehat{P}_T(\alpha) \equiv \int_{-\infty}^{\infty} e^{-\alpha k} P_T(k) dk, \tag{15}$$

and now change the order of integration, taking $\zeta_0 = 1$ with no loss of generality:

$$\begin{aligned}
\widehat{P}_T(\alpha) &= \int_{-\infty}^{\infty} e^{-\alpha k} E\left[\zeta_T \left(e^k - e^s\right)^+\right] dk \\
&= E\ \zeta_T \int_{-\infty}^{\infty} e^{-\alpha k} \int_s^{\infty} I_{\{y \leq k\}} e^y dy dk \\
&= E\ \zeta_T \int_s^{\infty} e^{-(\alpha-1)y}\ dy/\alpha \\
&= E\ \zeta_T\ \exp(-(\alpha-1)s)/\alpha(\alpha-1) \\
&= E[\zeta_T S_T^{1-\alpha}]/\alpha(\alpha-1) \\
&= \delta_0^{1-\alpha} e^{-\rho T} E\big[\, \delta_T^{1-\alpha-R} v(\xi_T)^{1-\alpha} \big| \delta_0 = 1 \,\big]/\alpha(\alpha-1) \\
&= \delta_0^{1-\alpha} e^{-\rho T} E\big[\, e^{(1-\alpha-R)(\int_0^T \sigma(\xi_s)dW_s + \int_0^T \tilde{\mu}(\xi_s)ds)} v(\xi_T)^{1-\alpha} \big| \delta_0 = 1 \,\big]/\alpha(\alpha-1) \\
&= \delta_0^{1-\alpha} e^{-\rho T} E\big[\, e^{\int_0^T z_\alpha(\xi_s)ds} v(\xi_T)^{1-\alpha} \big| \delta_0 = 1 \,\big]/\alpha(\alpha-1) \\
&= \delta_0^{1-\alpha} e^{-\rho T} \exp(T(Q + Z_\alpha)) v^{1-\alpha}/\alpha(\alpha-1),
\end{aligned}$$

where

$$z_\alpha(x) \equiv (1-\alpha-R)\widetilde{\mu}(x) + \frac{1}{2}(1-\alpha-R)^2\sigma^2(x),$$

and Z_α is the diagonal matrix of z_α.

We used the method of Hosono [Ho] (see also Abate-Whitt [AW]) to numerically invert the Laplace transform of the put price (15). We then compared the results obtain with the B&S formula in case of constant dividend yield. Results are quite encouraging. The speed of computation using the Laplace transform method is broadly comparable with that of the analytic B&S formula.

[2] As in this example, or more commonly for log-Lévy price processes.

4 Calibration methodology

In order to understand how well the model we are working with might fit data, we took some put option prices on the S&P500, and tried a number of different fitting procedures. This section presents the calibration methods used, and the next presents and discusses the numerical results obtained.

The data set consisted of put option prices written on the S&P500 (obtained from Bloomberg) referring to five consecutive trading days from February 23th, 2004 to February 27th, 2004. The choice of this particular index was motivated by its liquidity and the availability of option data for a reasonable set of expiries and strikes.

For each day under consideration, the set of data consisted of 12 strikes ranging from 950 to 1200 (on average approximately -17% to +6% of the at the money strike), and 3 expiries: 18 September 04, 18 December 04 and 18 June 05. We shall indicate by $\widehat{P}^n$ the set of data observed on day n. For each n thus, $\widehat{P}^n$ is a 12×3 matrix whose ij^{th} entry is the price of put option with strike K_i and expiry M_j. We will denote by $P(\theta, x)$, the 12×3 matrix of model prices for parameter vector θ, given that the Markov chain is in state x. Explicitly, θ contains the coefficient of relative risk aversion R, the agent's discount factor ρ, the elements of the Q-matrix Q, and the vectors μ and σ from the model specification (1).

As a first attempt, we tried a day by day calibration using a two state Markov chain. In order to find the set of parameters θ_n which best fitted market data on day n, we minimized the Average Relative Percentage Error (ARPE)[3]

$$L(\theta_n) = \frac{1}{N}\sum_{ij} \frac{\left|\widehat{P}^n_{ij} - \left(\sum_{x=1}^{S} \pi^n_x P(\theta_n, x)\right)_{ij}\right|}{\widehat{P}^n_{ij}}, \tag{16}$$

where N is the total number of market prices considered in each day of the calibration, S the number of possible states of the Markov chain and π^n_x denotes the weight assigned to state x of the chain on day n. The ARPE criterion gives the average loss per dollar invested if we used the model prices instead of the market prices.

As it is apparent from (16), we are comparing observed prices with a weighted average of the prices stemming from our model. The rationale behind this is

[3]The calibration was performed using also other quality of fit measure such as the Average Absolute Error as a percentage of the mean price (APE), the Average Absolute Error (AAE) and the Root Mean Square Error (RMSE). An mathematical definition of these measure is given in [SST] . The results obtained minimizing these loss functions are comparable to those obtained using ARPE, so we do not report them.

the following: the model requires knowledge of the current state of the Markov chain in order to produce a unique price for the option, and this state will in general not be observable. We propose therefore that the market price should be some average over the 'pure' prices obtained if the state of the Markov chain were known with certainty. Indeed, Arbitrage Pricing Theory tells us that market prices should be expectations (under the pricing measure) of discounted payoffs, and these expectations may be computed by firstly conditioning on the current state of the Markov chain, then averaging over the possible states of the chain; this explains the assumed form.

In calibrating the model thus, we have to mix over all possible states of the chain on each given day. To select the weights used, we should ideally use the conditional distribution (under the pricing measure) of the current state of the chain given all observations to date. In the absence of a long run of data to give a meaningful estimate of this distribution, we simply treat the weights as unknowns to be optimised over. This approach turned out to be the most effective and produced quite satisfactory results.

One of the main objections to the day-by-day calibration method is that the estimated parameters θ_n depend on n, so are not constant over the calibration period. However, as Table 1 shows, parameters estimated using this method were fairly stable over the (short) time interval. To increase parameter stability, we modified (16) by including a penalty for inter-day changes in the parameters. More precisely, we set

$$LS(\theta_n) = L^n(\theta_n) + \beta \parallel (\theta_n - \theta_{n-1})^2 \parallel . \tag{17}$$

where β is an arbitrary positive constant. A high β, will increase the stability of the parameter set but will worsen the fit to market data. In general we found that the stability of θ_n did not significantly improve by using (17) instead of (16), and the related results will be omitted.

So far we have only included option prices in the calibration, ignoring interest rates or equivalently bond prices. However, once we specify the parameter vector θ, we have uniquely determined the vector of interest rates consistent with the model. In other words, there exists only one piecewise constant, right-continuous, short-rate process $r(\xi)$ consistent with the model assumptions and it is given by

$$r(\xi) = \rho + \mu(\xi)R - \frac{1}{2}\sigma(\xi)^2 R(R+1); \tag{18}$$

see (9). Once the value of θ is chosen, the bond prices are given by

$$\begin{aligned} B^{t,T}(\theta, x) &= E_t \left[\exp\left(\int_t^T r(\xi_u) du \right) \right] \\ &= \left(\exp\left[(T-t)\left(Q - \mathbf{R} \right) \right] \mathbf{1} \right)(x), \end{aligned}$$

where $\mathbf{R}$ is obtained by diagonalizing the vector r. Function (16) can be easily modified to include bond prices. The new quality of fit measure will be given by

$$LB(\theta_n) = \frac{L(\theta_n)}{2} + \frac{\gamma}{2N_B}\sum_T \frac{\left|\widehat{B}^{n,T} - \sum_{x=1}^{N} \pi_x^n B^{n,T}(\theta_n, x)\right|}{\widehat{B}^{n,T}}, \tag{19}$$

where γ is an arbitrary constant measuring the relative weight of bond prices in the calibration, $\widehat{B}^{n,T}$ is the market price of a US T-bill or T-bond on day n with maturity T, and N_B is the number of bond prices used in the calibration. In particular we tried to fit the short-term end of the US treasury curve by using the 3, 6, 12 month T-bill and the 2 year T-bond.

We also calibrated the model using a 3 state state Markov-chain. However, the quality of fit was not better than the one obtained using a 2 state chain, and the estimation procedure was significantly slower. We thus omit these results.

5 Numerical Results

We shall now present some of the numerical results obtained using the calibration methodologies discusses in the previous section. Table 1 shows the value of the calibrated parameter set θ when weights are chosen "optimally", i.e. they are included in the parameter set and no bond is included in the calibration.

Table 1: Parameter estimates

	Day 1	**Day 2**	**Day 3**	**Day 4**	**Day 5**
μ_1	0.0560	0.0631	0.0626	0.0617	0.0582
μ_2	0.4048	0.3822	0.3454	0.3361	0.3716
σ_1	0.0990	0.1045	0.1031	0.1043	0.1064
σ_2	0.4619	0.4613	0.4448	0.4440	0.4581
Q_{12}	0.92	0.91	0.91	0.92	0.87
Q_{21}	3.43	3.43	3.43	3.43	3.43
ρ	0.0900	0.0900	0.0941	0.1049	0.1245
R	2.523	2.523	2.541	2.541	2.509
π_1	0.9134	0.9253	0.9189	0.9219	0.9584
$ARPE$	0.00545	0.00462	0.00551	0.00501	0.00475

Note that the average relative percentage price error is roughly of the order of 0.5%. In words, the model is able to fit the volatility surface skew quite closely. As a check we compared the implied volatility surface obtained from market prices with the one obtained using our model prices. To measure the goodness of fit of the surface, we used the following function

$$L_\sigma = \frac{1}{N} \sum_{ij} \left| \widehat{\sigma}_{ij}^n - \sigma_{ij}^n \right| .$$

The error so calculated was in the order of 6 bps, which is well inside the bid-ask spread for vanilla options on the S&P500. Also, estimated parameters appear to be fairly stable over time, even when we do not include any penalty for deviating from the previous day estimate.

Table 2 shows the parameter estimates when bonds are included in the calibration for both the approaches highlighted in the previous paragraph. As one would expect, trying to fit the interest rate curve and the volatility surface simultaneously leads to larger errors than in the 'vanilla' cases. However, the quality of fit is still acceptable.

Table 2: Parameter estimates including bonds

	Day 1	**Day 2**	**Day 3**	**Day 4**	**Day 5**
μ_1	0.0100	0.0100	0.0101	0.0100	0.0100
μ_2	0.0100	0.0106	0.0100	0.0143	0.0100
σ_1	0.0274	0.0299	0.0340	0.0345	0.0337
σ_2	0.2655	0.2572	0.2470	0.2482	0.2408
Q_{12}	0.3916	0.3603	0.3469	0.3466	0.3712
Q_{21}	3.4136	2.8707	2.6272	2.5873	2.5920
ρ	0.0900	0.0900	0.0900	0.0900	0.0900
R	4.3911	4.3038	4.3539	4.3397	4.3504
π_1	0.8382	0.8323	0.8325	0.8271	0.8274
$ARPE$	0.0145	0.0137	0.0138	0.0134	0.0146

The quality of fit is summarised in Figure 1, expressed in terms of the implied volatilities. Notice that this was not the criterion used to achieve the fit; nevertheless, the quality of fit is quite satisfying.

6 Comparison with other Models

In order to get a better grasp of how well our equilibrium model is able to fit the volatility skew, we compared our model to some of the standard models available in the literature. In particular the models we used as benchmarks are the Heston model, the Heston model with jumps, the Barndorff-Nielsen-Shepard model, the Variance-Gamma model with CIR and OU stochastic clock and the Normal Inverse Gaussian model with CIR and OU stochastic clock.

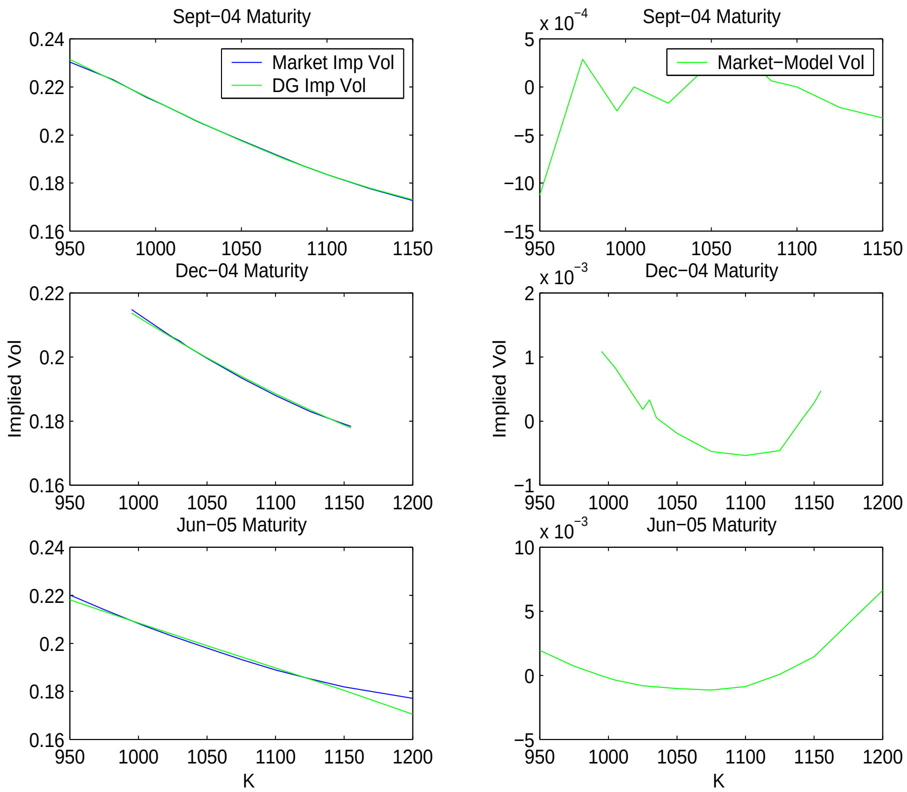

Figure 1: Summary of the quality of fit for each of the three expiries.

We did not perform the calibration of these model ourself, but we used instead some result obtained by [SST] and [CGMY] in two similar calibration exercise on the Eurostoxx 50 index and the SP500 respectively. Table 3 shows that our equilibrium model performs well with respect to all the above mentioned models; in fact it is the one with the lowest ARPE when bonds are not included in the calibration and it continues to perform well even when the Treasury curve is included in the estimation procedure. Of course, differences may be attributable as much to the use of different calibration datasets as to properties of the models, but without common calibration data there is nothing that can be done about this.

Table 3: Average Relative Percentage Error across models

HEST	**HESTJ**	**BNS**	**VGCIR**	**VGOU**
0.0174	0.0126	0.0221	0.0106	0.0190
NIGCIR	**NIGOU**	**CMGYSA**	**DR**	**DR-B**
0.0099	0.0175	0.0206	0.0051	0.014

7 Conclusions

We have presented a simple equilibrium model where the single production activity in the economy pays a continuous stochastic dividend whose dynamics follows a Markov-modulated geometric Brownian motion. Such a model may offer one explanation for the observed phenomenon of jumps in stock prices. The model also allows for stochastic drift and volatility in the dynamics of the stock. We showed how to price European-style vanilla options by Laplace transform methods. In terms of computational speed, this method turned out to be very fast. Calibration to market data required some care, but even using a two state Markov chain, the model is able to fit the volatility surface quite closely.

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