DIVERSE BELIEFS

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Overview

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- Different forms of diversity
- Private information (PI) equilibria
- Diverse beliefs (DB) equilibria
- Main result: when is a PI equilibrium a DB equilibrium?
- Conclusions

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- Diverse beliefs include diverse information!

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Bars denote variables in PI setting

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Tildes denote variables in diverse beliefs problem

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THEOREM. Suppose that $(\bar{S}, \bar{\Theta}, \bar{C})$ is a PI equilibrium with initial allocation $y \in \mathbb{R}^J$ for the discrete-time finite-horizon Lucas tree model introduced above. Then it is possible to construct a filtered measurable space $(\tilde{\Omega}, (\tilde{\mathcal{G}}_t)_{t \in \mathbb{T}})$, carrying $\tilde{\mathcal{G}}$-adapted processes $\tilde{X}, \tilde{S}, \tilde{\Theta}, \tilde{C}$ of dimensions $d, 1, J$ and J respectively, and probability measures P^j, $j = 1, \ldots, J$, on $(\tilde{\Omega}, \tilde{\mathcal{G}}_T)$ such that $(\tilde{S}_t, \tilde{\Theta}_t, \tilde{C}_t)_{t \in \mathbb{T}}$ is a DB equilibrium with initial allocation on $y \in \mathbb{R}^J$ and beliefs $(P^j)_{j=1}^J$ with the property that

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- A PI equilibrium is observationally indistinguishable from a DB equilibrium;

Main result.

THEOREM. Suppose that $(\bar{S}, \bar{\Theta}, \bar{C})$ is a PI equilibrium with initial allocation $y \in \mathbb{R}^J$ for the discrete-time finite-horizon Lucas tree model introduced above. Then it is possible to construct a filtered measurable space $(\tilde{\Omega}, (\tilde{\mathcal{G}}_t)_{t \in \mathbb{T}})$, carrying $\tilde{\mathcal{G}}$-adapted processes $\tilde{X}, \tilde{S}, \tilde{\Theta}, \tilde{C}$ of dimensions $d, 1, J$ and J respectively, and probability measures P^j, $j = 1, \ldots, J$, on $(\tilde{\Omega}, \tilde{\mathcal{G}}_T)$ such that $(\tilde{S}_t, \tilde{\Theta}_t, \tilde{C}_t)_{t \in \mathbb{T}}$ is a DB equilibrium with initial allocation on $y \in \mathbb{R}^J$ and beliefs $(P^j)_{j=1}^J$ with the property that

$$\mathcal{L}(X, \bar{S}, \bar{\Theta}, \bar{C}) = \mathcal{L}(\tilde{X}, \tilde{S}, \tilde{\Theta}, \tilde{C}).$$

- $(\tilde{\Omega}, (\tilde{\mathcal{G}}_t)_{t \in \mathbb{T}})$ does not in general have any private signals;
- A PI equilibrium is observationally indistinguishable from a DB equilibrium;
- .. so we gain no modelling advantage by working with (complicated) PI models .... !

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- ... but even if I think your signals are uninformative, I cannot ignore them, because you rely on them in your choices;

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- ... but even if I think your signals are uninformative, I cannot ignore them, because you rely on them in your choices; and that makes them informative.

Outline of proof.

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Agent j in PI equilibrium chooses optimal $(\bar{\theta}^j, \bar{c}^j)$.

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Agent j in PI equilibrium chooses optimal $(\bar{\theta}^j, \bar{c}^j)$. His state-price density $\bar{\lambda}_t^j = U_j'(t, \bar{c}_t^j)$ must have the property

$$\bar{\lambda}_t^j \bar{S}_t = E\left[\bar{\lambda}_{t+1}^j (\bar{S}_{t+1} + \delta_{t+1}) \,\middle|\, \bar{\mathcal{F}}_t^j\right]. \qquad \text{(PI-FOC)}$$

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How do we go from the first to the second??

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- Change (PI-FOC) to

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where $\mathcal{F}_t^j \subset \bar{\mathcal{F}}_t^j$ depends only on public information relating to agent j;

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- Enlarge $\mathcal{F}_t^j$ to include all public information, by specifying (independent) distribution for other agent's variables.

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Leading-order change to objective is

$$\begin{aligned} E\Big[\sum_{t=0}^T U'_j(t,\bar c^j_t)\epsilon_t\Big] &= E\Big[\sum_{t=0}^T \bar\lambda^j_t\{\eta_t(\bar S_t+\delta_t) - \eta_{t+1}\bar S_t\}\Big] \\ &= E\Big[\sum_{t=1}^T \eta_t(\bar\lambda^j_t(\bar S_t+\delta_t) - \bar\lambda^j_{t-1}\bar S_{t-1})\Big] \\ &= E\Big[\sum_{t=1}^T \eta_t E(\bar\lambda^j_t(\bar S_t+\delta_t) - \bar\lambda^j_{t-1}\bar S_{t-1} \,\big|\, \bar{\mathcal{F}}^j_{t-1})\Big], \end{aligned}$$

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where $\mathcal{F}^j_t \equiv \sigma(X_u, \bar S_u, \bar\theta^j_{u+1}, \bar c^j_u \;:\; u \le t) \subseteq \sigma(X_u, \bar S_u, \bar\theta^j_{u+1}, \bar c^j_u, z^j_u \;:\; u \le t) = \bar{\mathcal{F}}^j_t$.

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- Under P^j, $(\tilde{X}, \tilde{\theta}^j, \tilde{c}^j) \sim P^*$;
- Conditional on $(\tilde{X}, \tilde{\theta}^j, \tilde{c}^j)$, law of $\tilde{S}$ is $\kappa^j(\tilde{X}, \tilde{\theta}^j, \tilde{c}^j; \cdot)$;

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Then:

- P^j-distribution of $(\tilde{X}, \tilde{\theta}^j, \tilde{c}^j, \tilde{S})$ is the P-distribution of $(X, \bar{\theta}^j, \bar{c}^j, \bar{S})$;

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- Expand to $\tilde{\Omega} \equiv \Omega_0 \times \mathbb{R}^{T+1}$, and set ($\tilde{\omega} = (\omega, s)$, $\omega \in \Omega_0$)

$$\tilde{X}(\tilde{\omega}) = X(\omega), \quad \tilde{\Theta}(\tilde{\omega}) = \Theta(\omega), \quad \tilde{C}(\tilde{\omega}) = C(\omega), \quad \tilde{S}_t(\tilde{\omega}) = s_t$$

where $s = (s_0, \ldots, s_T) \in \mathbb{R}^{T+1}$. Set $\tilde{\mathcal{G}}_t = \sigma(\tilde{X}_u, \tilde{\Theta}_{u+1}, \tilde{C}_u, \tilde{S}_u : u \leq t)$.

Define P^j on $\tilde{\Omega}$:

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- Conditional on $(\tilde{X}, \tilde{\theta}^j, \tilde{c}^j)$, law of $\tilde{S}$ is $\kappa^j(\tilde{X}, \tilde{\theta}^j, \tilde{c}^j; \cdot)$;
- Conditional on $(\tilde{X}, \tilde{\theta}^j, \tilde{c}^j, \tilde{S})$, the $\tilde{\theta}^i, \tilde{c}^i$, $i \neq j$ are chosen independently subject to the constraints $\sum \tilde{\theta}^i_t = 1$, $\sum \tilde{c}^i_t = \delta_t$.

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Uses:

Proposition. If X is an integrable random variable, if $\mathcal{G}$ and $\mathcal{A}$ are two sub-σ-fields of $\mathcal{F}$ such that $\mathcal{A}$ is independent of X and $\mathcal{G}$, then

$$E[X|\mathcal{G}] = E[X|\mathcal{G} \vee \mathcal{A}] \qquad \text{a.s..}$$

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Then

$$\begin{aligned}
E^j \sum_{t=0}^{T} U_j(t, c_t) &\leq E^j \sum_{t=0}^{T} \Big[U_j(t, \tilde{c}^j_t) + \tilde{\lambda}^j_t (c_t - \tilde{c}^j_t) \Big] \\
&= E^j \sum_{t=0}^{T} \Big[U_j(t, \tilde{c}^j_t) + \tilde{\lambda}^j_t \Big\{ (\theta_t - \tilde{\theta}^j_t)(\tilde{S}_t + \tilde{\delta}_t) - (\theta_{t+1} - \tilde{\theta}^j_{t+1})\tilde{S}_t \Big\} \Big] \\
&= E^j \sum_{t=0}^{T} U_j(t, \tilde{c}^j_t) + E^j \sum_{t=1}^{T} (\theta_t - \tilde{\theta}^j_t) \Big\{ \tilde{\lambda}^j_t (\tilde{S}_t + \tilde{\delta}_t) - \tilde{\lambda}^j_{t-1} \tilde{S}_{t-1} \Big\} \\
&= E^j \sum_{t=0}^{T} U_j(t, \tilde{c}^j_t)
\end{aligned}$$

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